ML-guided and Robust Constraint Acquisition

Dimos Tsouros, Hendrik ‘Henk’ Bierlee, Senne Berden, Tias Guns

2025-08-11 - Progress Towards The Holy Grail

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DTAI
Declarative Languages & Artificial Intelligence

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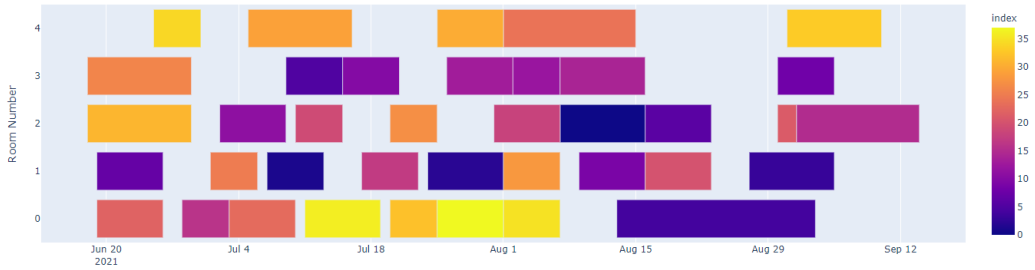
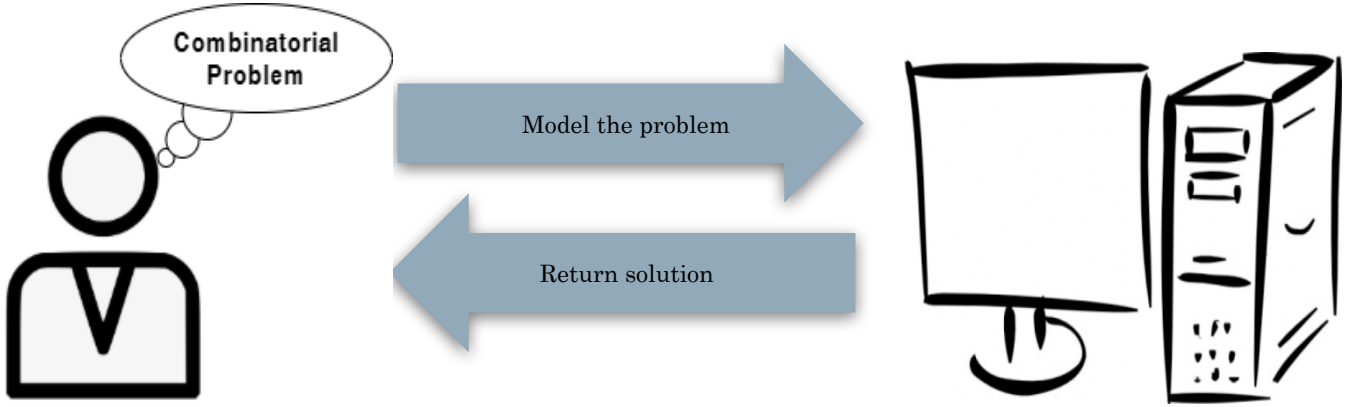
Introduction

- Constraint programming (CP)
  - Solving combinatorial problems in AI

	Week 1							Week 2							Total shifts
	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun	
name															
Megan	-	D	D	D	D	-	-	D	D	-	-	D	D	D	9
Katherine	D	D	D	D	D	-	-	-	D	D	-	-	D	D	9
Robert	D	D	D	-	-	D	D	D	-	-	D	D	-	-	8
Jonathan	D	D	-	-	-	D	D	D	D	D	-	-	-	-	7
William	-	D	D	D	D	-	-	D	D	-	-	D	D	D	9
Richard	D	D	D	-	-	-	D	D	D	D	-	-	-	-	7
Kristen	-	-	D	D	D	-	-	D	D	-	-	D	D	D	8
Kevin	D	D	-	-	D	D	-	-	D	D	D	D	-	-	8
Cover D	5/5	7/7	6/6	4/4	5/5	3/5	3/5	6/6	7/7	4/4	2/2	5/5	4/6	4/4	14



- Model + Solve paradigm



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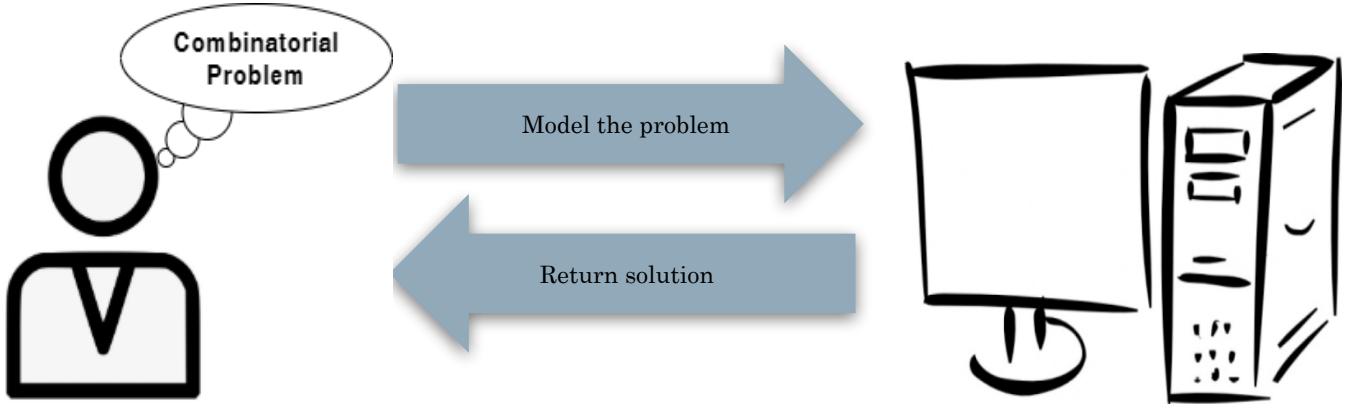

- Variables *X* with their domains *D*

```
# Variables
grid = cp.intvar(1, grid_size, shape=(grid_size, grid_size), name="grid")
```

- Constraints of the problem:

```
# Constraints on rows and columns
for row in grid:
    model += cp.AllDifferent(row)

for col in grid.T:  # numpy's Transpose
    model += cp.AllDifferent(col)

# Constraints on blocks
for i in range(0, grid_size, block_size_row):
    for j in range(0, grid_size, block_size_col):
        model += cp.AllDifferent(grid[i:i + block_size_row, j:j + block_size_col])
```

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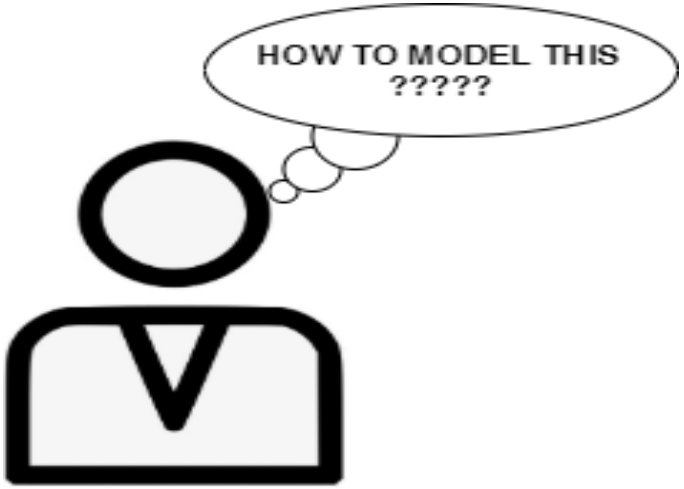
Introduction

Modelling is not always trivial

- Requires expertise
- Bottleneck for the wider use of CP



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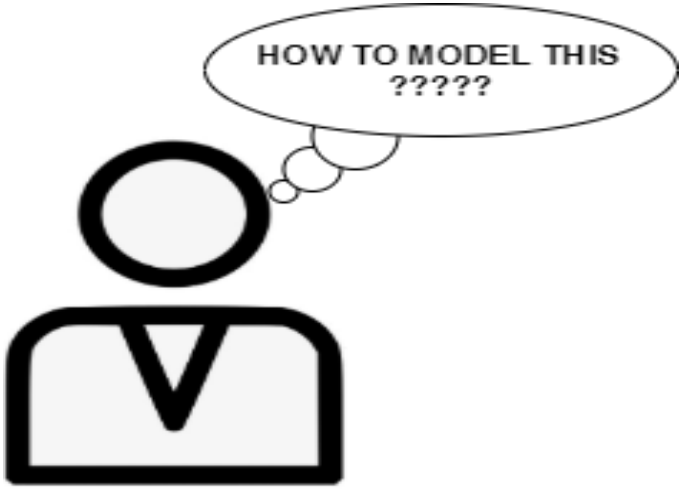
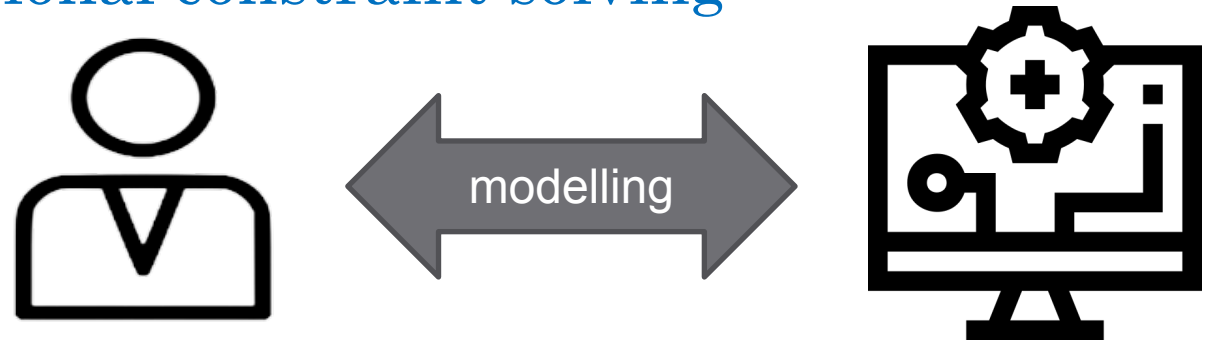


Constraint Acquisition:

- System and user modeling together!
- Conversational constraint solving



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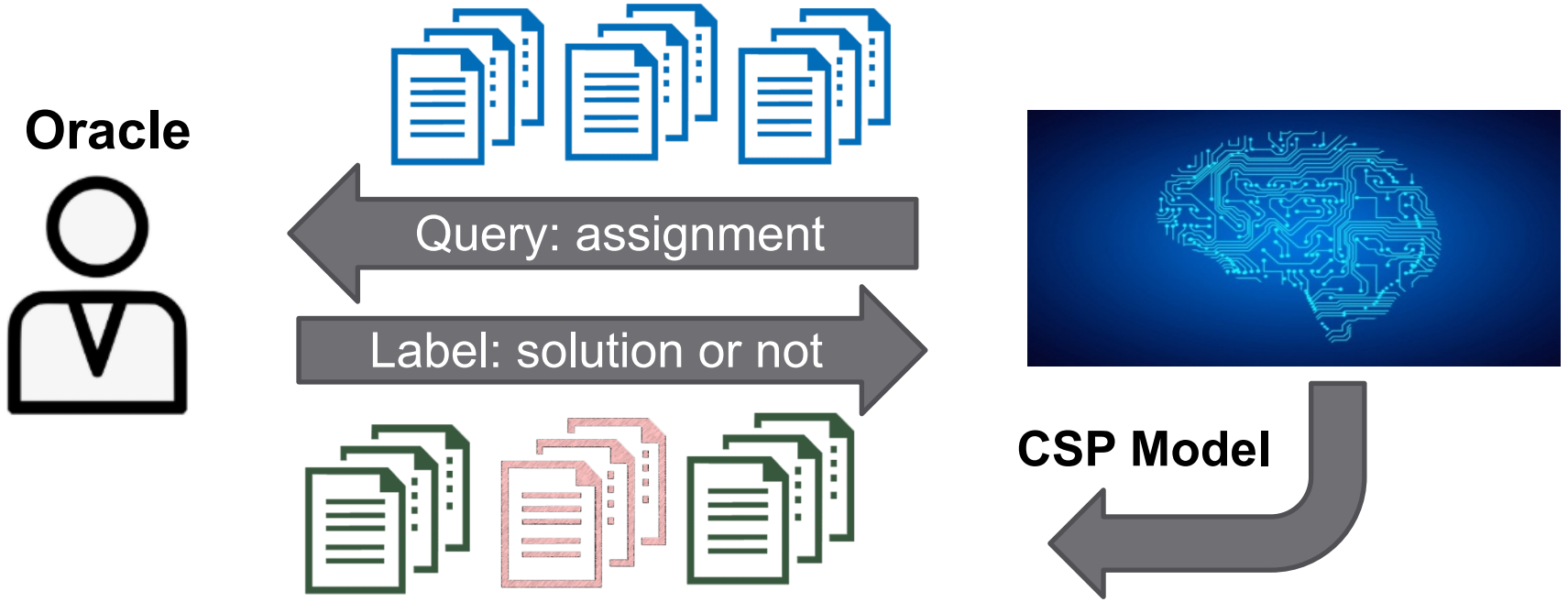
Interactive Constraint Acquisition



Learning Constraints through Partial Queries, C. Bessiere et al., AIJ, 2023

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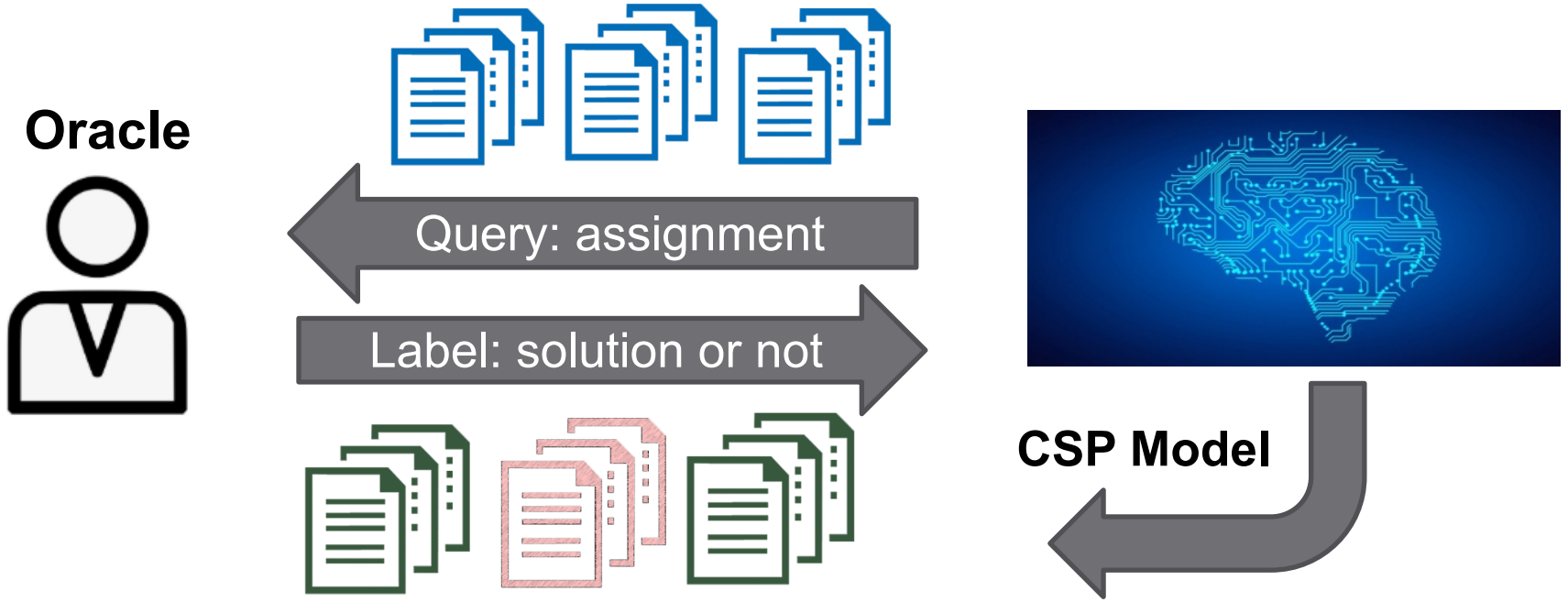

Membership query

1	1	3	4
3	2	1	1
2	2	3	1
2	3	4	3

Partial query

1	1	-	4
3	-	1	-
-	-	-	-
2	-	-	-

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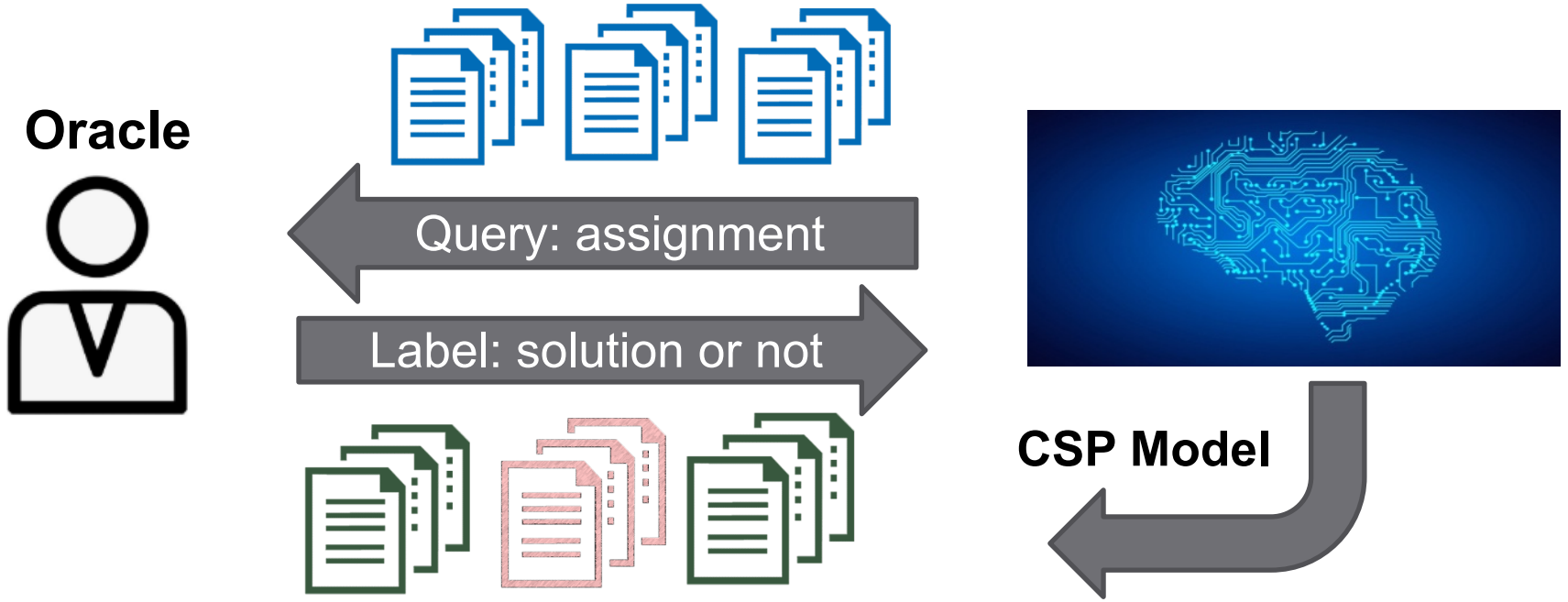

Membership query

1	1	3	4
3	2	1	1
2	2	3	1
2	3	4	3

Answer: "Non-solution" in both of them (a constraint is violated)

Partial query

1	1	-	4
3	-	1	-
-	-	-	-
2	-	-	-

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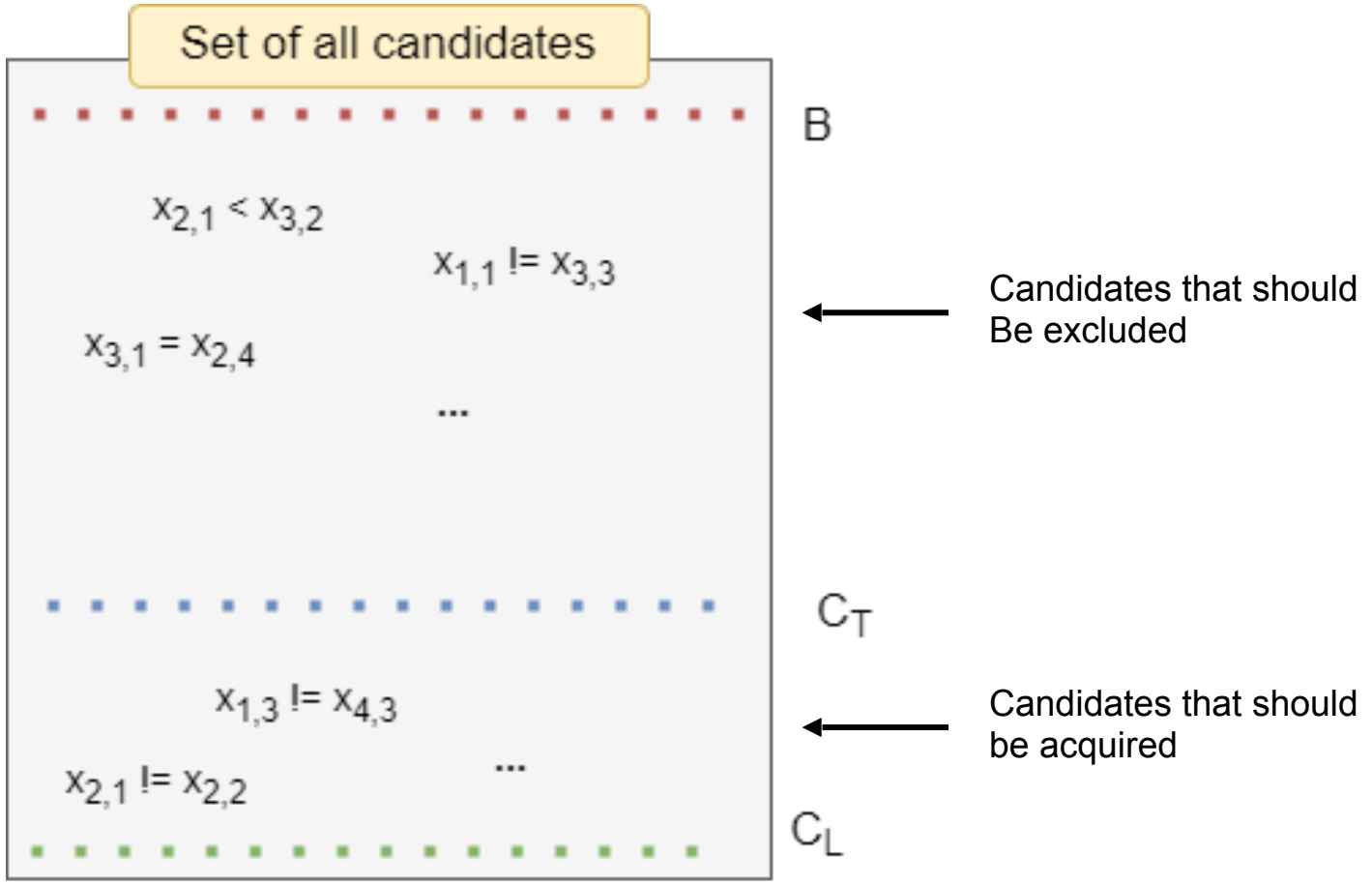
Candidate Elimination

- **B**: set of (remaining) candidate constraints
- $\mathbf{C_T}$: target set of constraints
- $\mathbf{C_L}$: learned set of constraints



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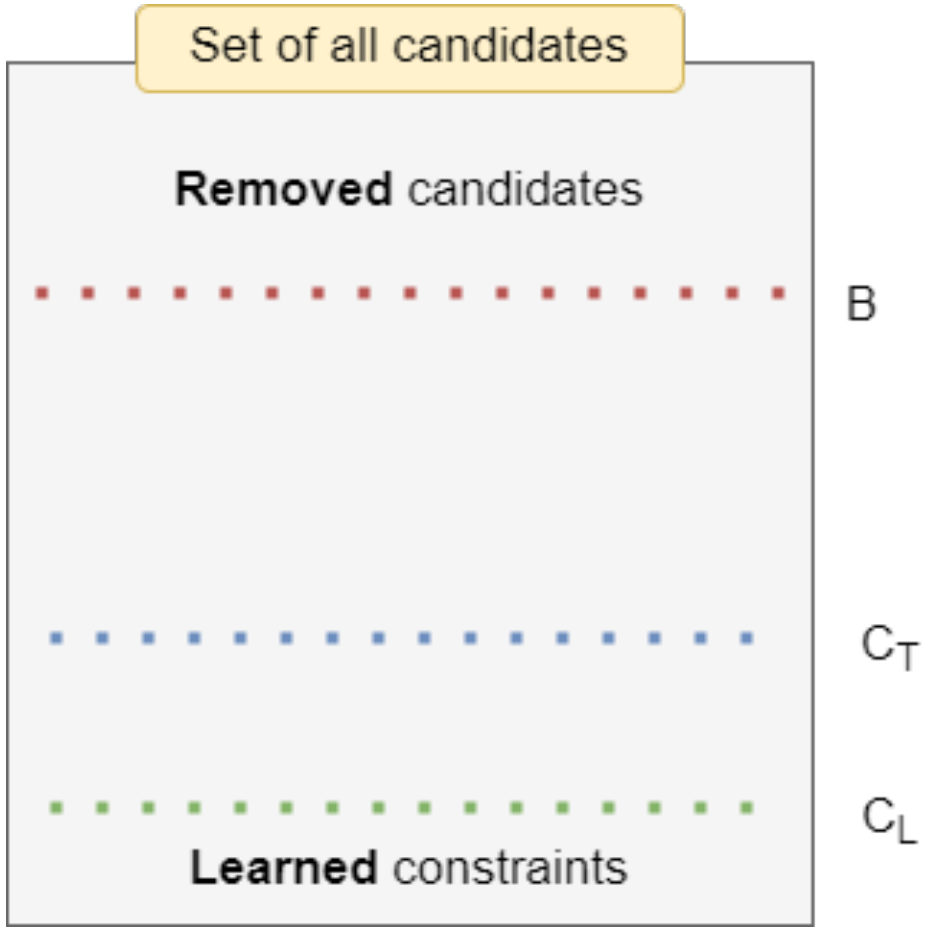
Candidate Elimination

During the learning process:

- Constraints are removed from B
- Constraints are added to C_L



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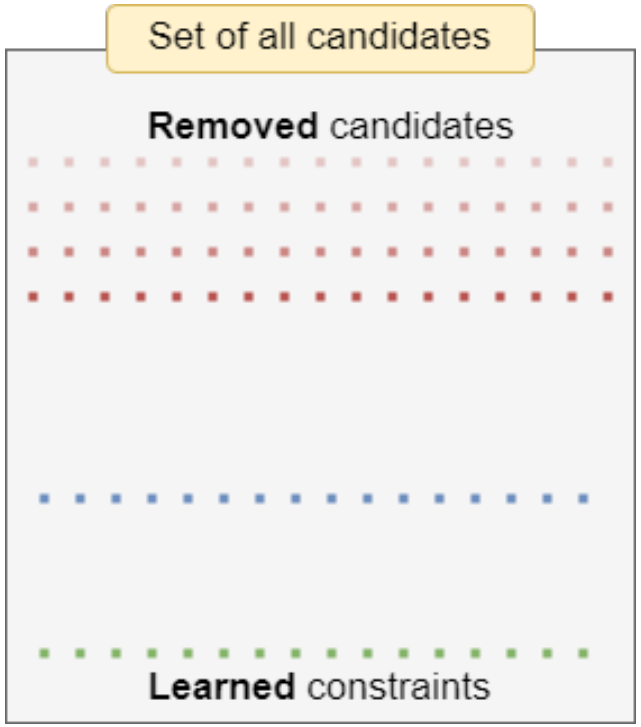
Learning from examples

- Examples: Assignments to the variables of the problem

- Learning from ***positive*** examples ("Solution"):

- Violated constraints cannot be part of the model
- Otherwise, it could not be a solution

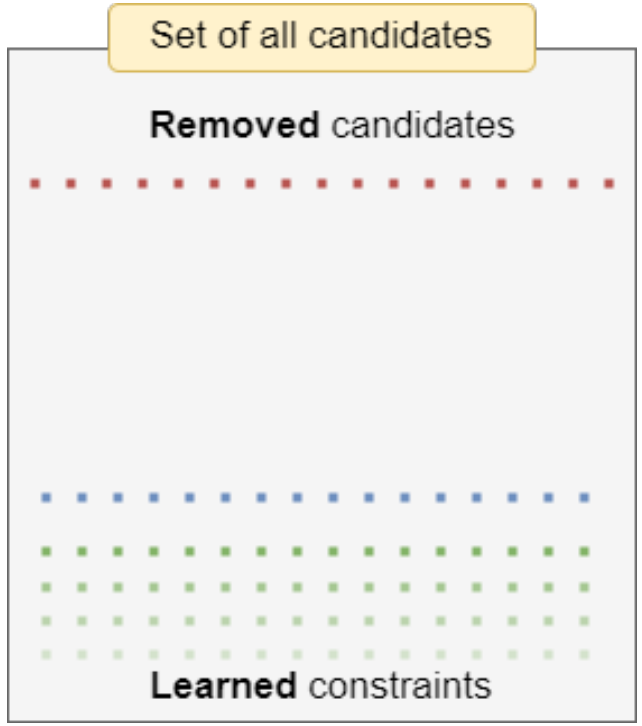
↳ Eliminating candidates



- Learning from ***negative*** examples ("Non-solution"):

- One (or more) violated constraint is a constraint of the problem
- Otherwise, it would be a solution

↳ Learning Constraints



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Generate example

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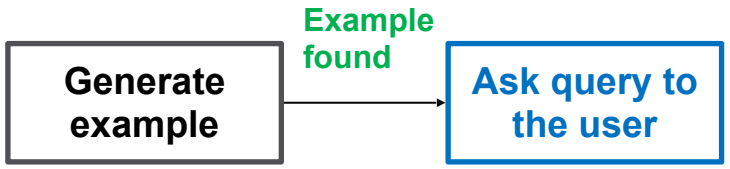

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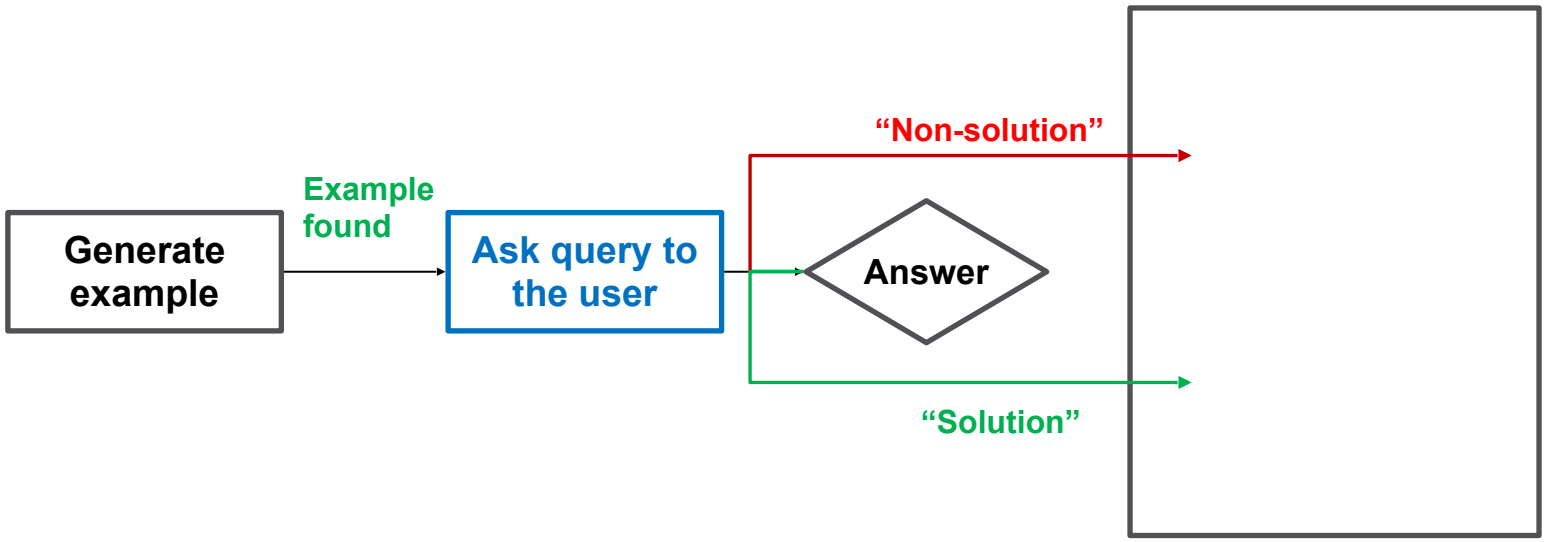

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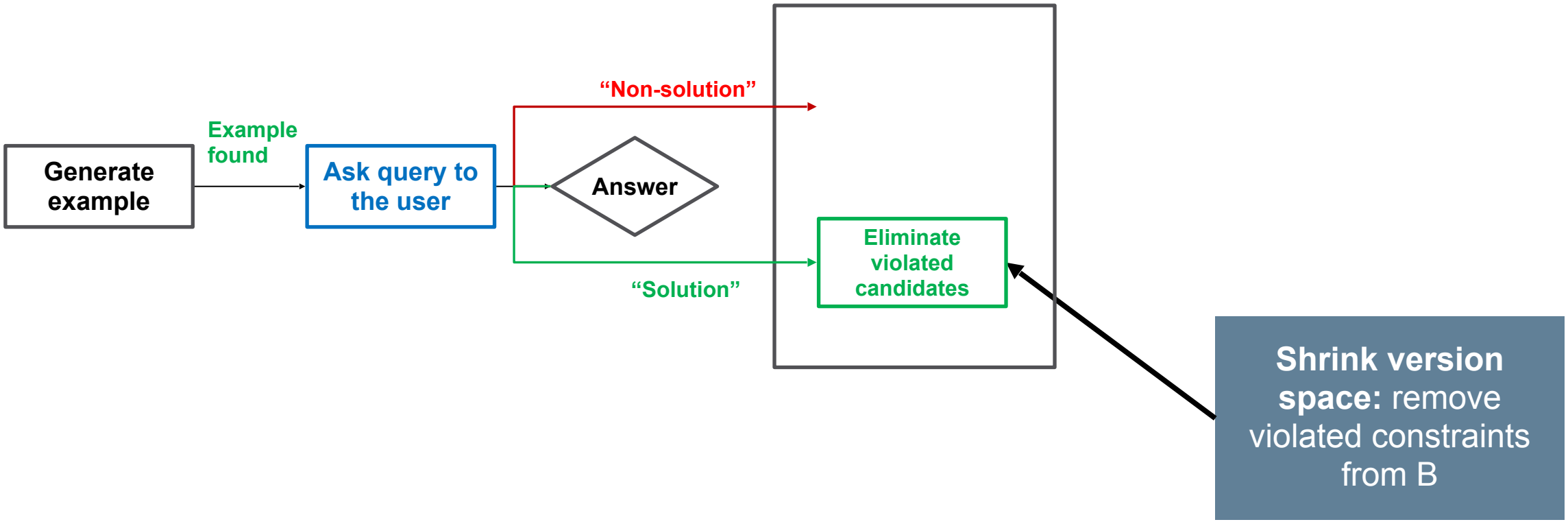

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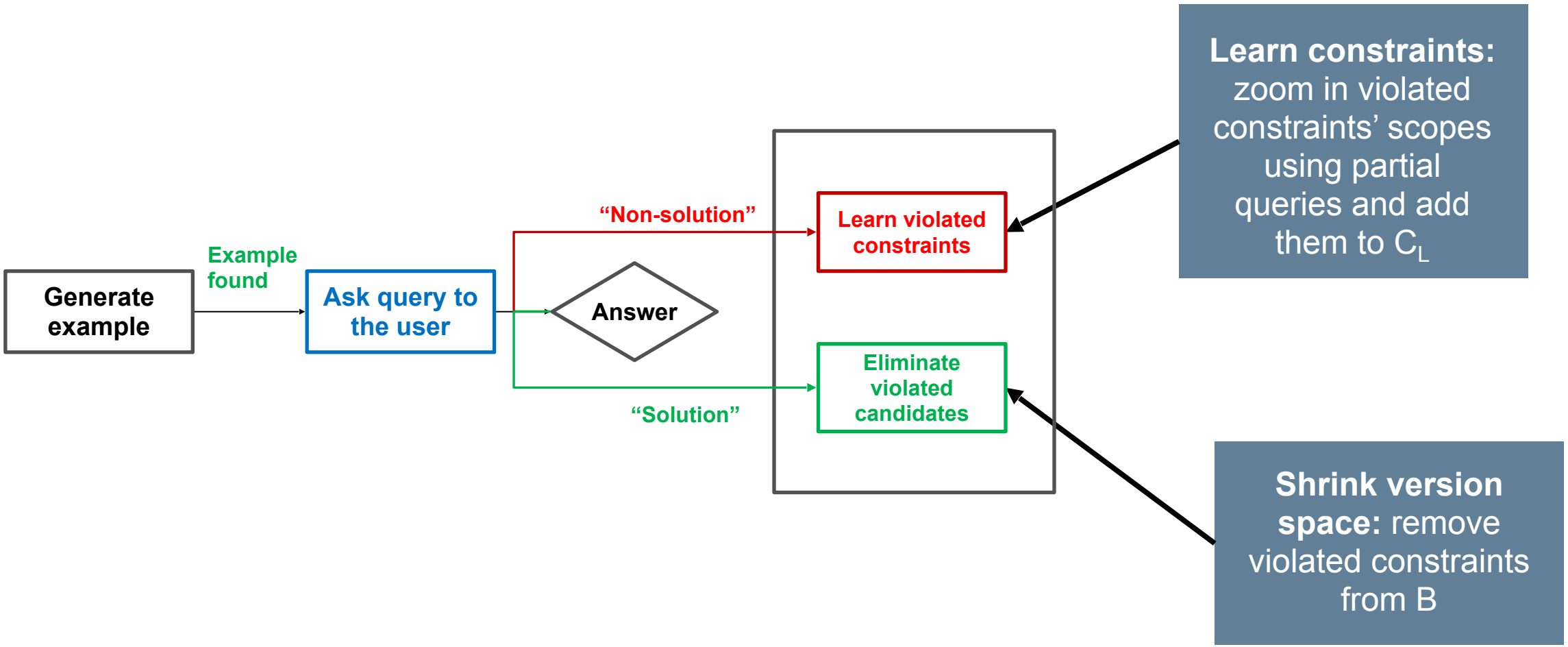

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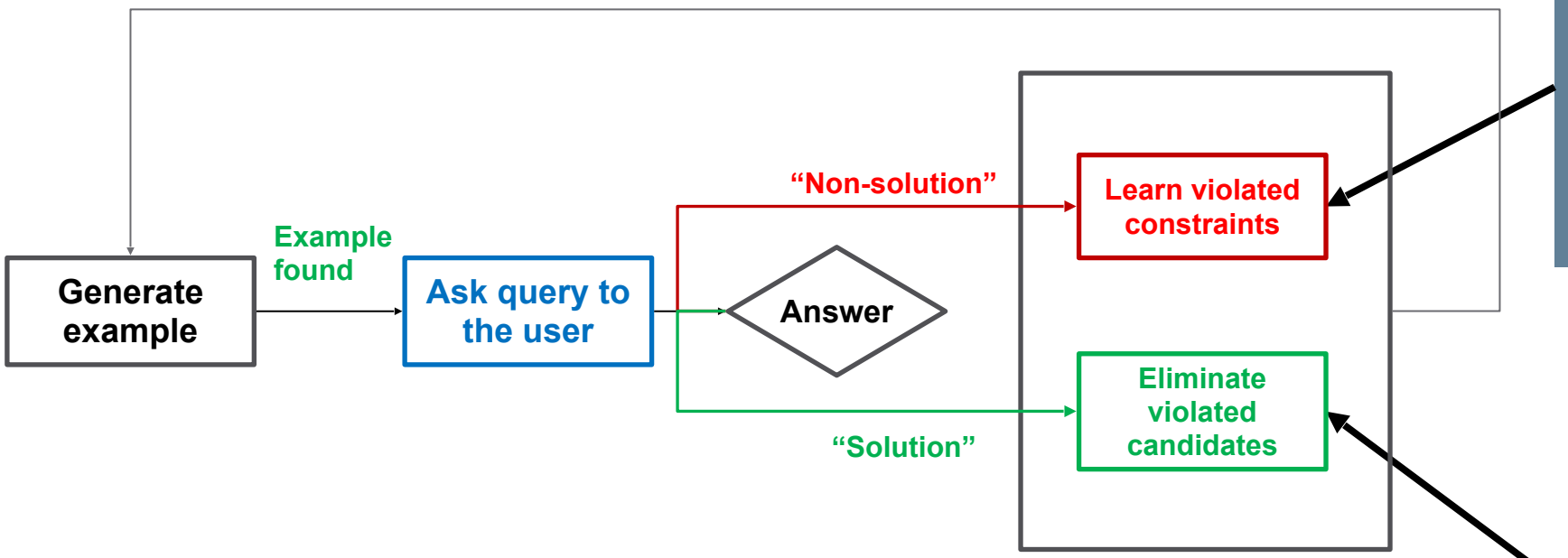

Learn constraints: zoom in violated constraints' scopes using partial queries and add them to C_L

Shrink version space: remove violated constraints from B

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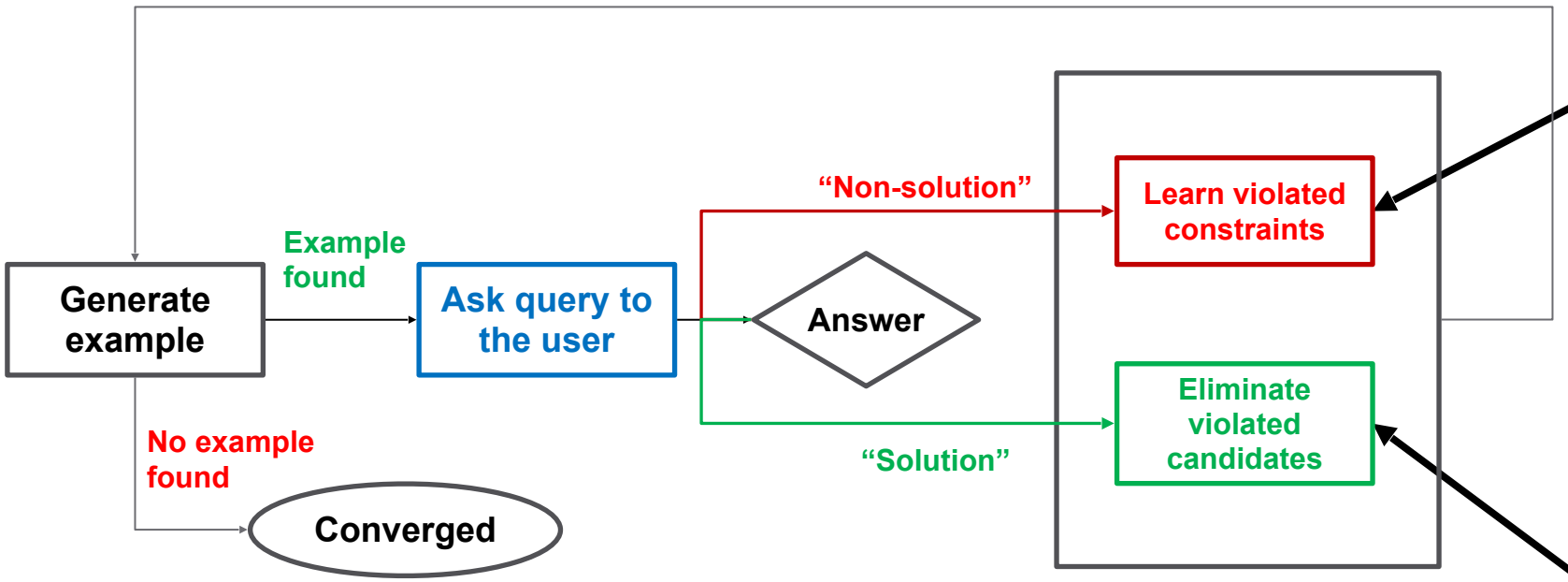

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PyConA

https://github.com/CPMpy/PyConA

pycona 0.3

```
pip install pycona
```

Using the CPMpy modeling library

https://github.com/CPMpy/CPMpy

Hands-on on interactive CA Demos

Multiple Acquisition

```
[ ]  # Multiple Acquisition: includes MquAcq and MQuAcq2
     mqa = MQuAcq()
     learned_instance = mqa.learn(instance, oracle=oracle)

     mqa2 = MQuAcq2()
     learned_instance = mqa2.learn(instance, oracle=oracle)
```

GrowAcq

```
# Creating an environment with the default settings
env = ActiveCAEnv()

# GrowAcq
ga = GrowAcq(env)
learned_instance = ga.learn(instance, oracle=oracle)

# we can compare the statistics:
out = pd.concat([a.env.metrics.short_statistics for a in [qa,mqa,mqa2,ga]])
out.index = ["quacq","mquacq","mquacq2","growacq"]
out
```

	CL	tot_q	top_lvl_q	tfs_q	tfc_q	avg_q_size	avg_gen_time	avg_t	max_t	tot_t	conv
quacq	34	196	38	142	16	6.6939	0.8317	0.4407	1.4866	86.3767	1
mquacq	34	161	63	30	68	2.7950	0.8812	0.0470	1.4936	7.5736	1
mquacq2	34	169	35	102	24	4.7988	0.5819	0.0454	1.2612	7.6738	1
growacq	34	179	64	84	31	4.5978	0.0544	0.0236	0.4465	4.2266	1

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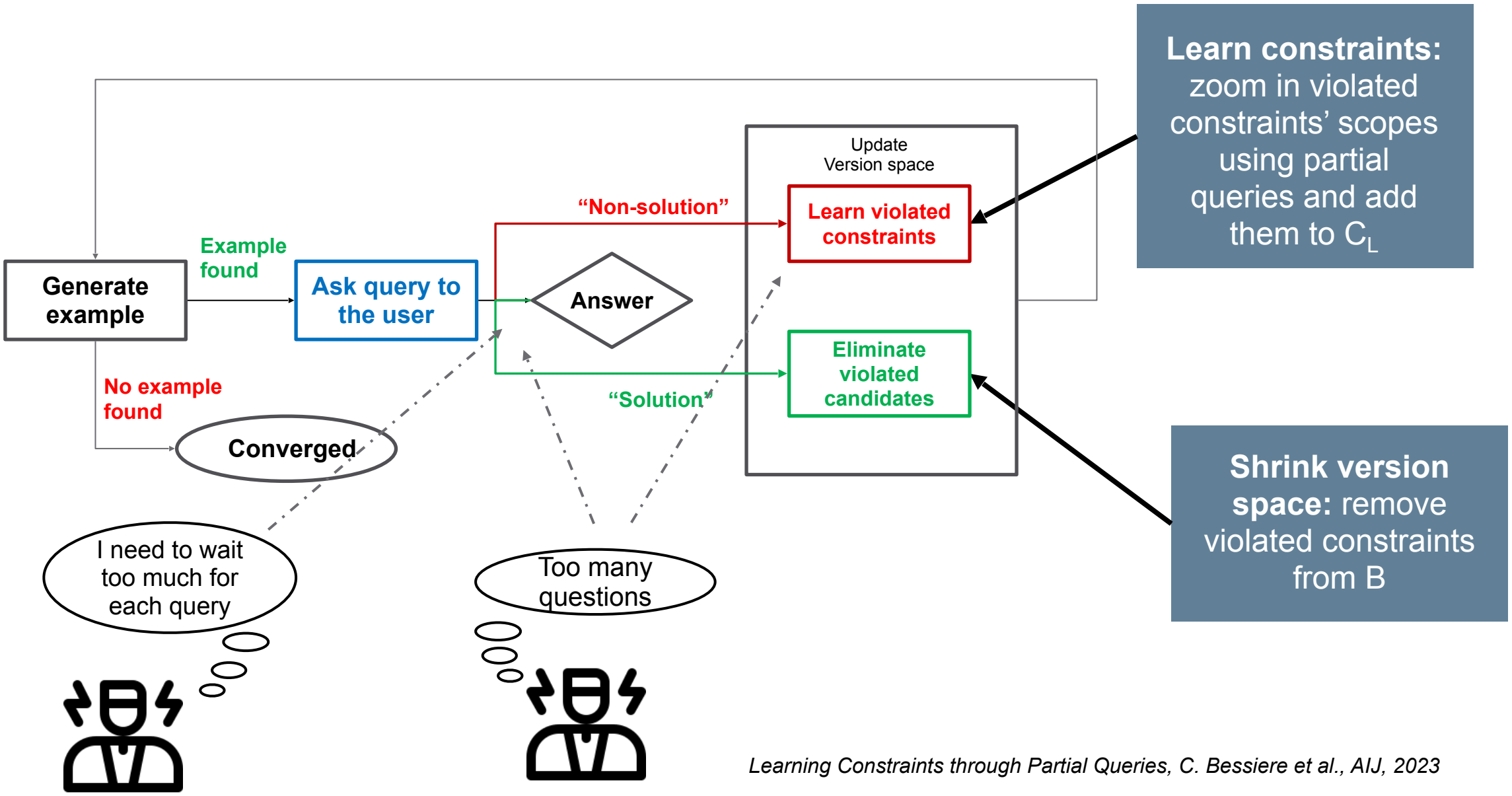

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Query generation

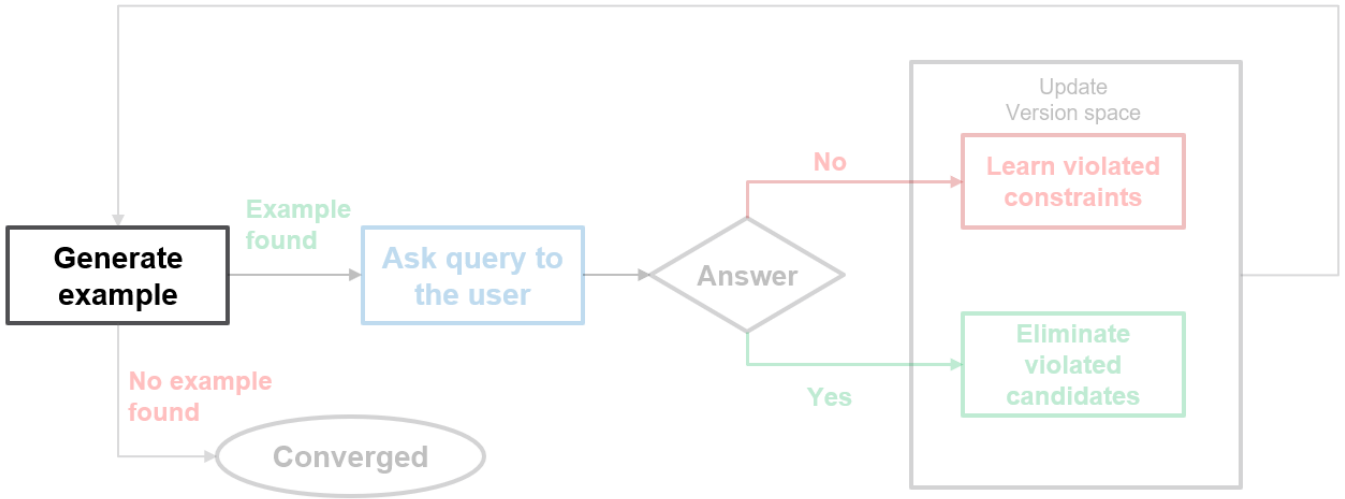
Informative query

- Generate “informative” examples
  - Solve a CSP

Quality of query

- Get the maximum amount of information
  - Solve a COP

Convergence



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Guiding Query Generation

Better generated examples lead to faster convergence

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Guiding Query Generation

Better generated examples lead to faster convergence

"Solution": Eliminate candidates fast

↓

The **more** candidates we have violated, the faster B will shrink

↓

$$max \sum_{c \in B} [\![e \notin sol(c)]\!]$$

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Guiding Query Generation

Better generated examples lead to faster convergence

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$$max \sum_{c \in B} [\![e \notin sol(c)]\!]$$

-"Non-solution": Find constraint(s) fast

The **less** candidates we have violated, the fewer additional queries needed to learn a constraint

$$min \sum_{c \in B} [\![e \notin sol(c)]\!]$$

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Opposite objectives, based on the (**future**) answer

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We cannot know the answer of the user before we ask the query → max violations

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We cannot know the answer of the user before we ask the query → max violations

The **more** candidates we have vi

The **less** candidates we have violated, the

But what if we can predict if a candidate is a constraint of the problem or not?

$$max \sum_{c \in B} [\![e \notin sol(c)]\!]$$

Opposite objectives, based on the (**future**) answer

$$min \sum_{c \in B} [\![e \notin sol(c)]\!]$$

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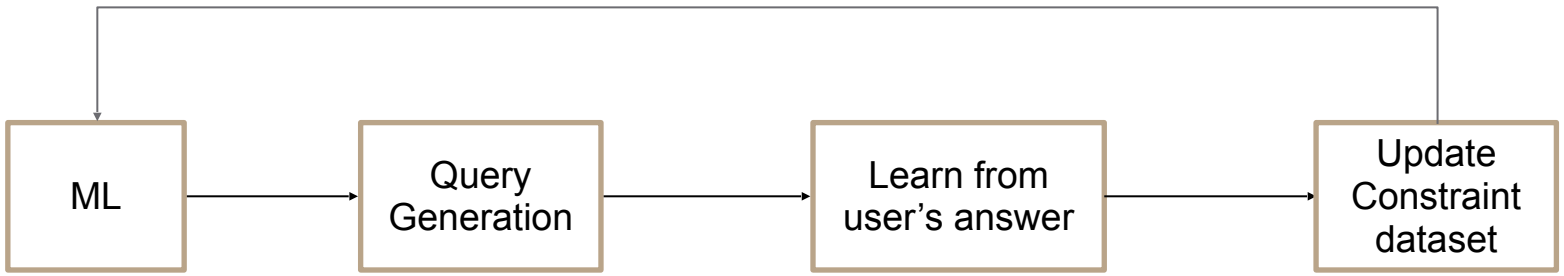
How to guide query generation?

Use a classidier $O(c)$ to predict if a candidate is a constraint of the problem or not

It is a prediction problem

Use Machine Learning!!



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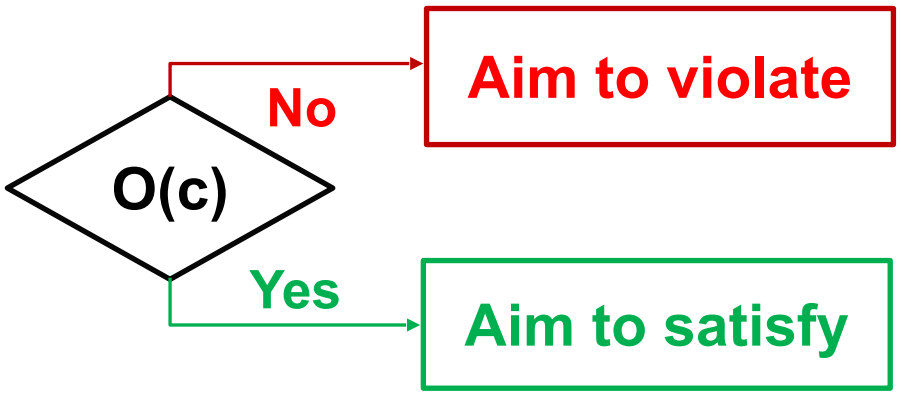

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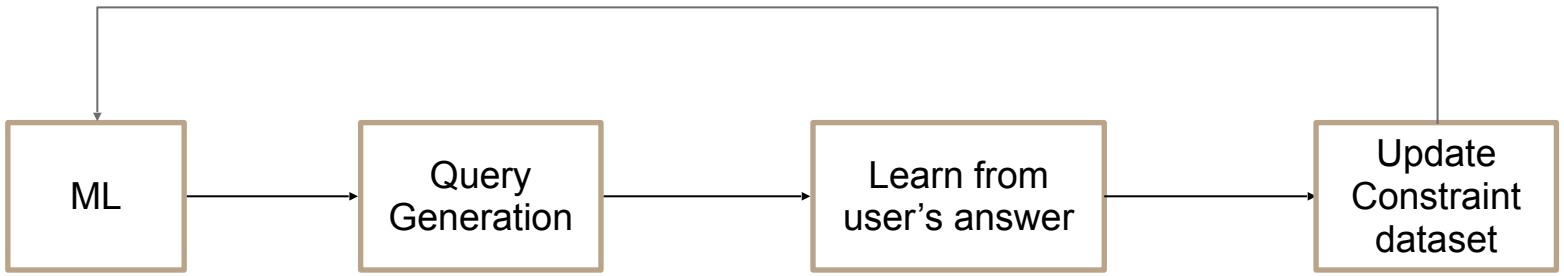

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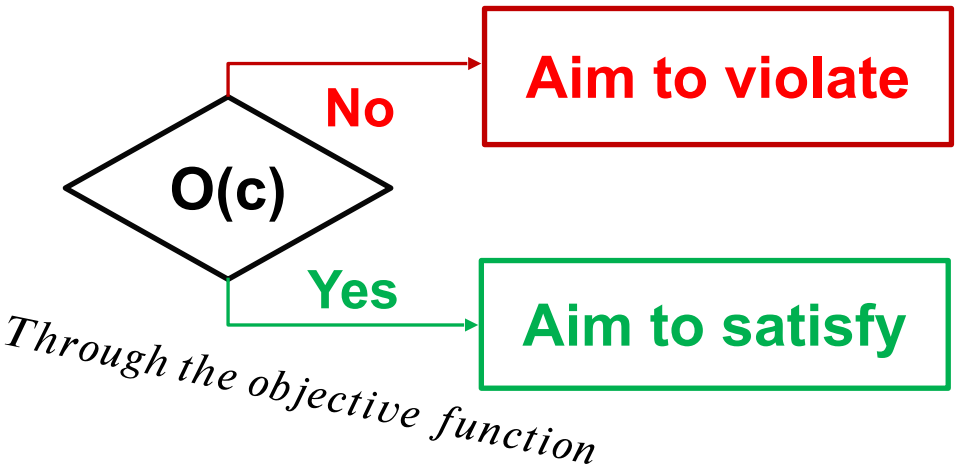

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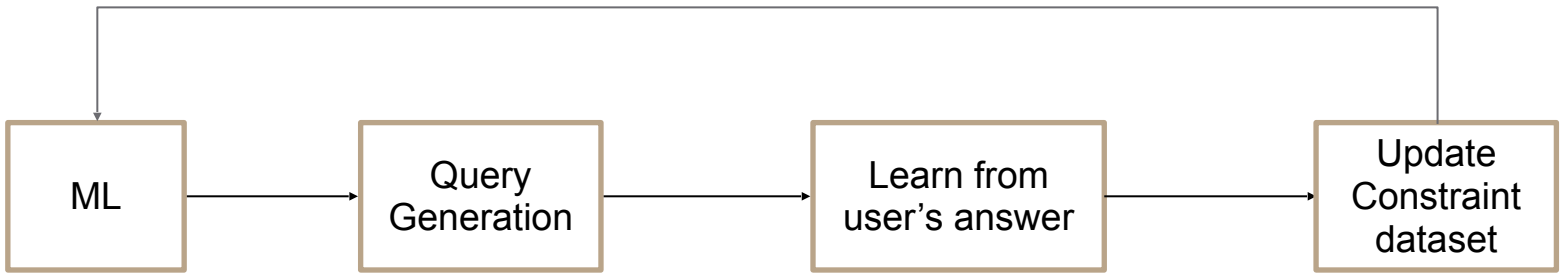

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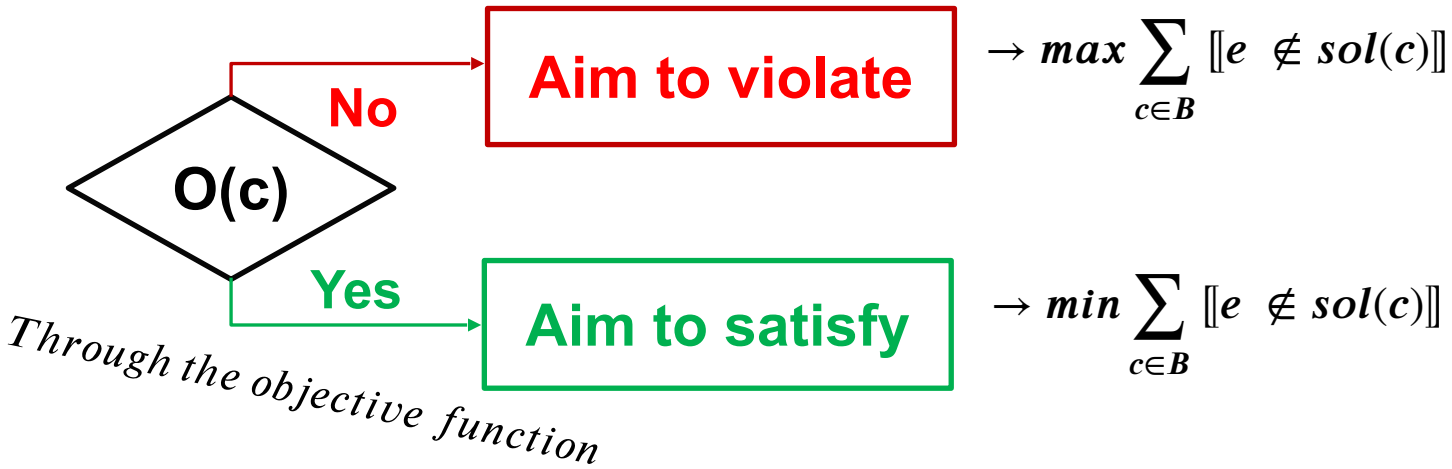

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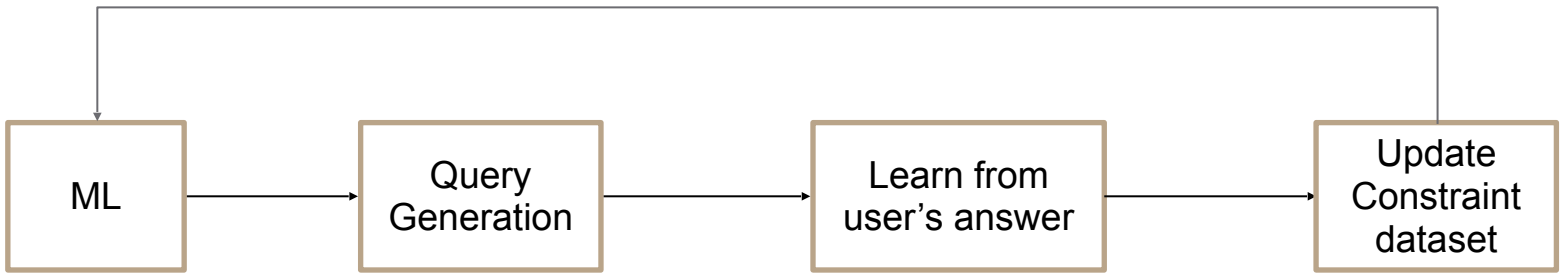

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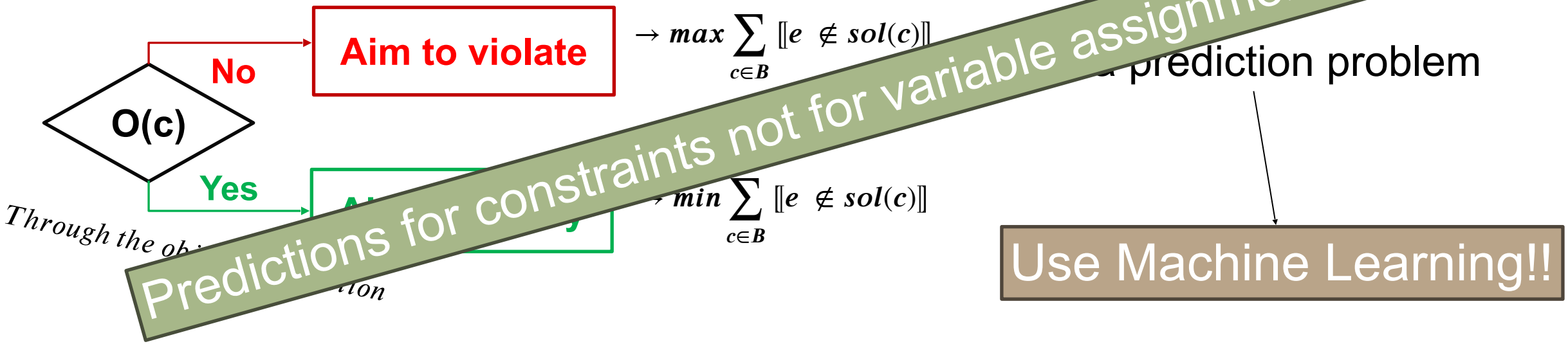

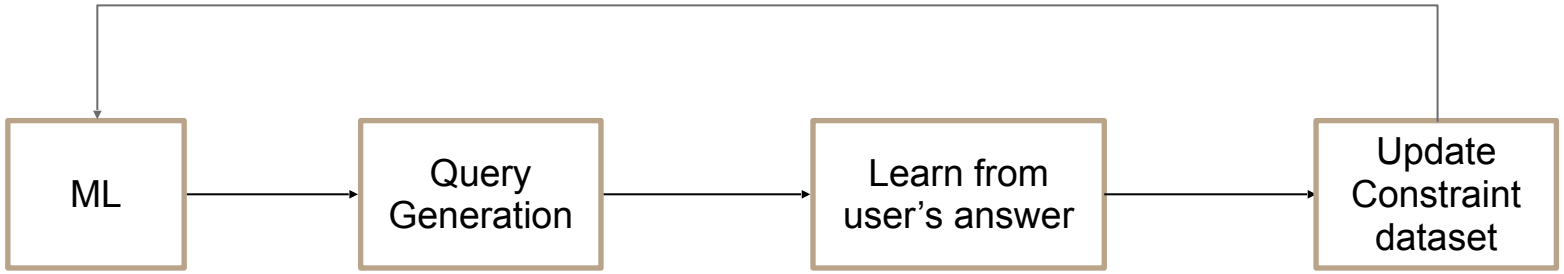

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Using Machine Learning for the prediction

Dataset: Constraint features and class (True or False)
-Constructing during the acquisition process
-Constraints that we know are part of the problem or not
-When a constraint is learned add a positive instance
-When a constraint is removed from *B add a negative instance*
-Use both relation and scope features

Relation-based features	Scope-based features
Relation name (string)	Dim[i] same_val (Bool)
Has constant (Bool)	Dim[i] avg (float)
Constant value (int)	Dim[i] distance (int)
Arity (int)	…

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Example for constraint
$x_{1,1} \neq x_{1,2}$ *in Sudoku*

Relation-based features		Scope-based features	
Relation name (string)	≠	Dim[i] same_val (Bool)	True, False
Has constant (Bool)	False	Dim[i] avg (float)	1, 1.5
Constant value (int)	-1	Dim[i] distance (int)	0, 1
Arity (int)	2	…	

For the 2 dimensions

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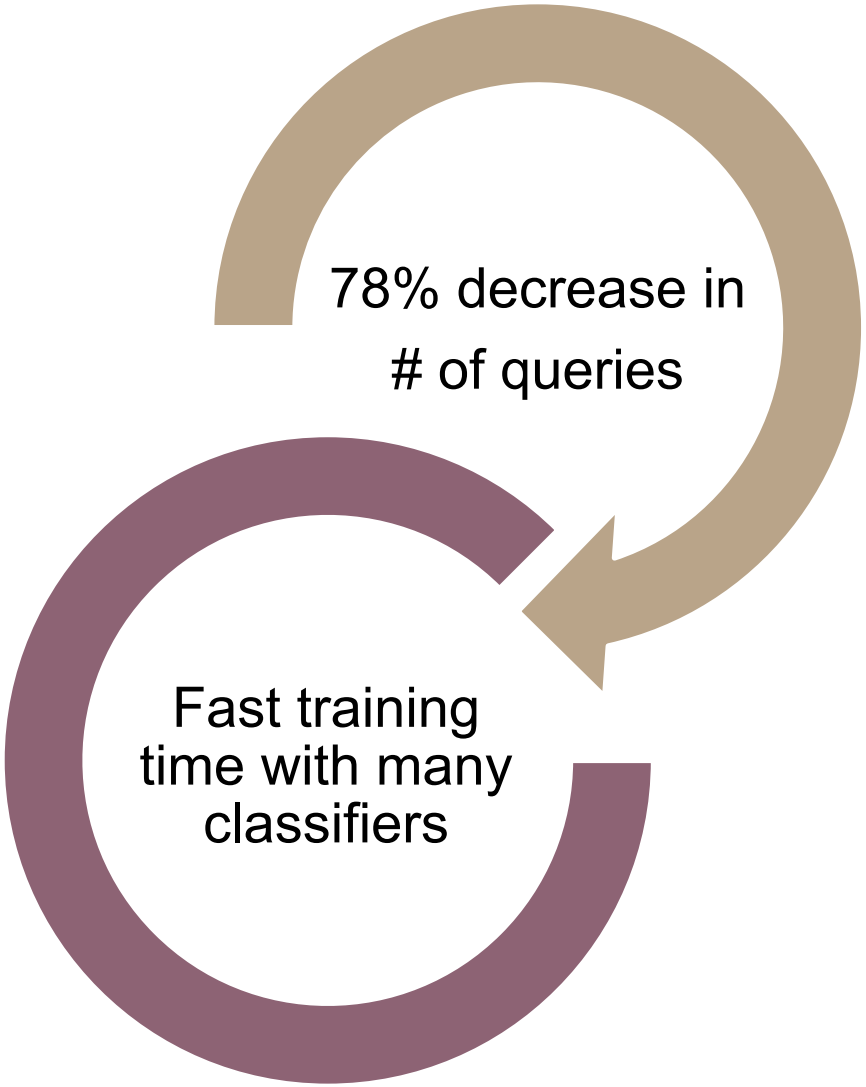
Does using ML to guide queries help?



Work in progress

Extract what classifier has learned and ask generalization queries

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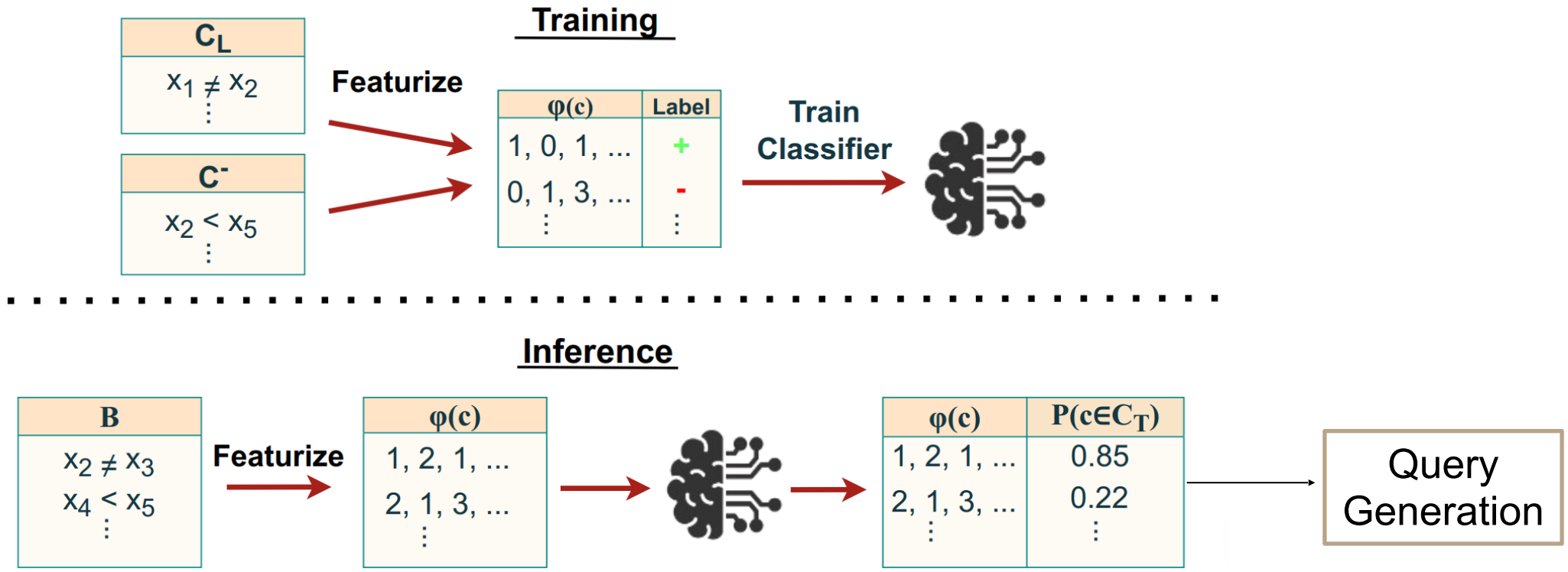
Learning to Learn in Interactive CA



Statistical ML learns the structure implicitly, query-based learning makes it explicit

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Major limitation

Assumption of always correct answers!

How to make
interactive constraint acquisition
robust?

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Motivation

New active CA methods require fewer and fewer queries

However, a single false query answer can lead to an unrecoverable state

To enable adoption, robustness against noise is required

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> “Uncertainty. How do we cope with imprecise, uncertain or noisy information?”
>
> – [E. C. Freuder]

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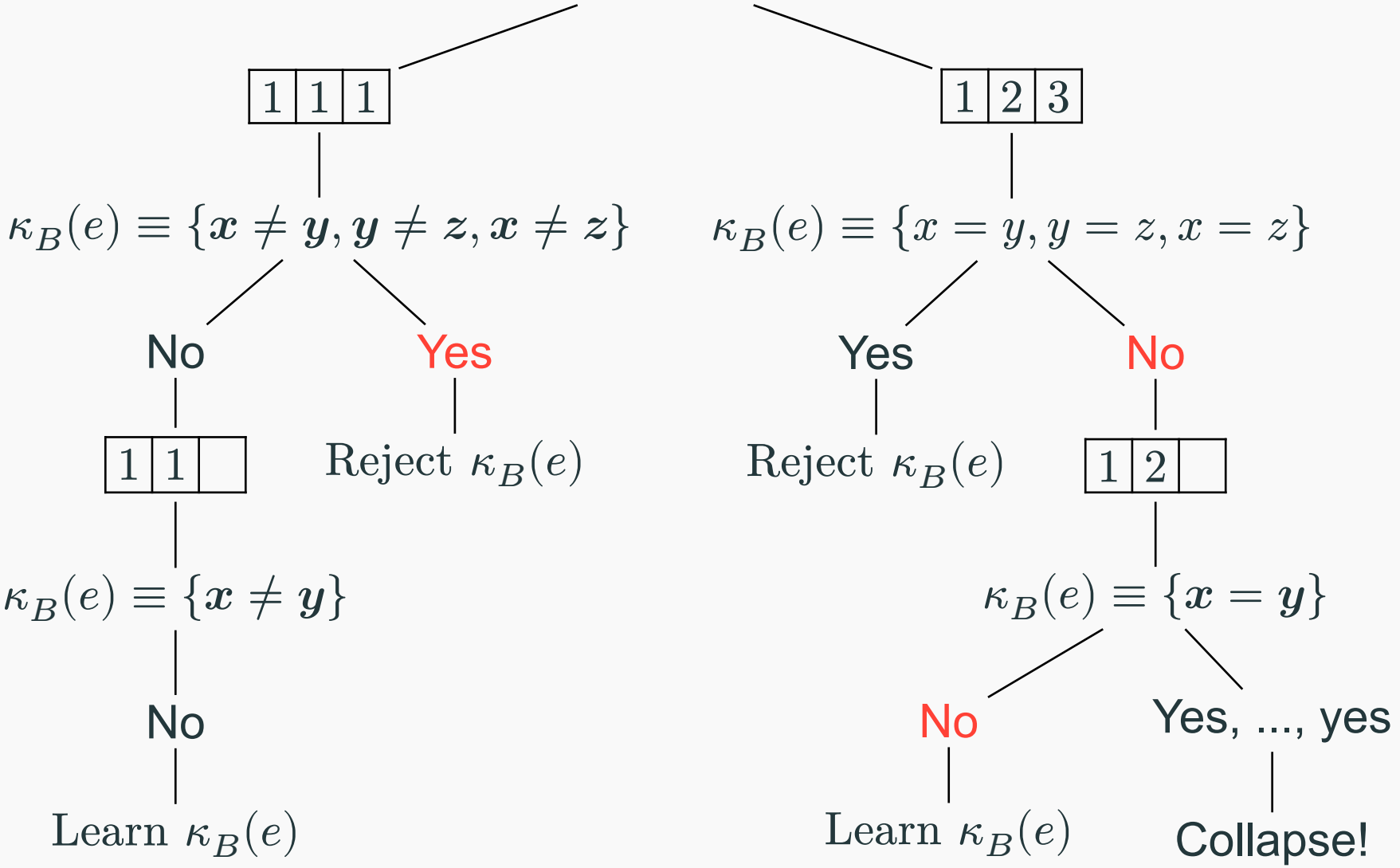
Example: AllDifferent(x, y, z) under noise

$C_T \equiv \{\boldsymbol{x \neq y}, \boldsymbol{y \neq z}, \boldsymbol{x \neq z}\}, B \equiv \{\boldsymbol{x \neq y}, \boldsymbol{y \neq z}, \boldsymbol{x \neq z}, x = y, y = z, x = z\}, C_L \equiv \{\}$



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What is robustness against noise?

Robust Active CA: acquire a C_L with a high probability of $C_L \leftrightarrow C_T$ under noise

Types of noise	**Example**
Uniform random	3% error rate
Based on query complexity	$\propto$ number of constraints the user needs to check (similar to [D. Angluin and D. K. Slonim])
Limited queries	Answering “I don’t know”

Robustness will come at a cost of extra queries and wait time

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Related work

In concept learning, there are robust methods using both membership **and** equivalence queries [L. Bisht, N. H. Bshouty, and L. Khoury]

In passive CA, the set of examples is fixed. Some passive CA methods are robust due to their statistical nature:

- `BayesAcq` [S. Prestwich]: Mimics a Naive Bayes classifier with a linear constraint
- `SeqAcq` [S. Prestwich]: All violated constraints of a negative example receive positive evidence (no sub-queries!)

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Robust active CA

Traditional active CA permanently removes constraints from B after one query

Instead, our idea is to store for each constraint c..

$$\mathrm{lrn}(c) \equiv \text{How many times have we learned } c?$$
$$\mathrm{rej}(c) \equiv \text{How many times have we rejected } c?$$

..then use $\mathrm{lrn}(c), \mathrm{rej}(c)$ to extract B and C_L from the initial bias B_0

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Extracting B and C_L

One measure of how "learned" a constraint c is:

$$\rho(c) \equiv \mathrm{lrn}(c) - \mathrm{rej}(c)$$

Extract B, C_L from initial bias B_0 by setting a bias interval $R \equiv [l..u]$:

$$c \in B_0 \text{ is } \begin{cases} \text{candidate } (c \in B) & \text{if } l \leq \rho(c) \leq u \\ \text{learned } (c \in C_L) & \text{if } \rho(c) > u \\ \text{rejected} & \text{if } \rho(c) < l \end{cases}$$

In other words:

$$B \equiv \{c : c \in B_0, l \leq \rho(c) \leq u\}, C_L \equiv \{c : c \in B_0, \rho(c) > u\}$$

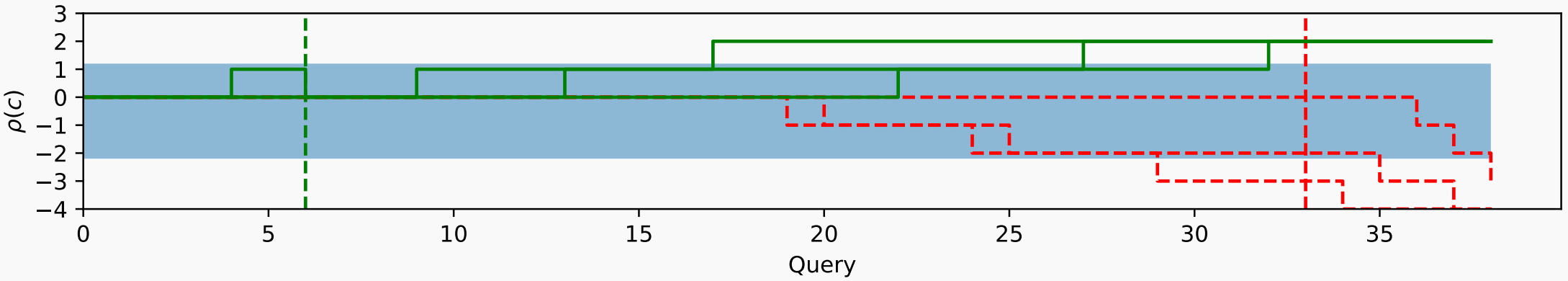

Figure 1: Learning $\mathrm{AllDifferent}(x, y, z)$ with one false positive and one false negative

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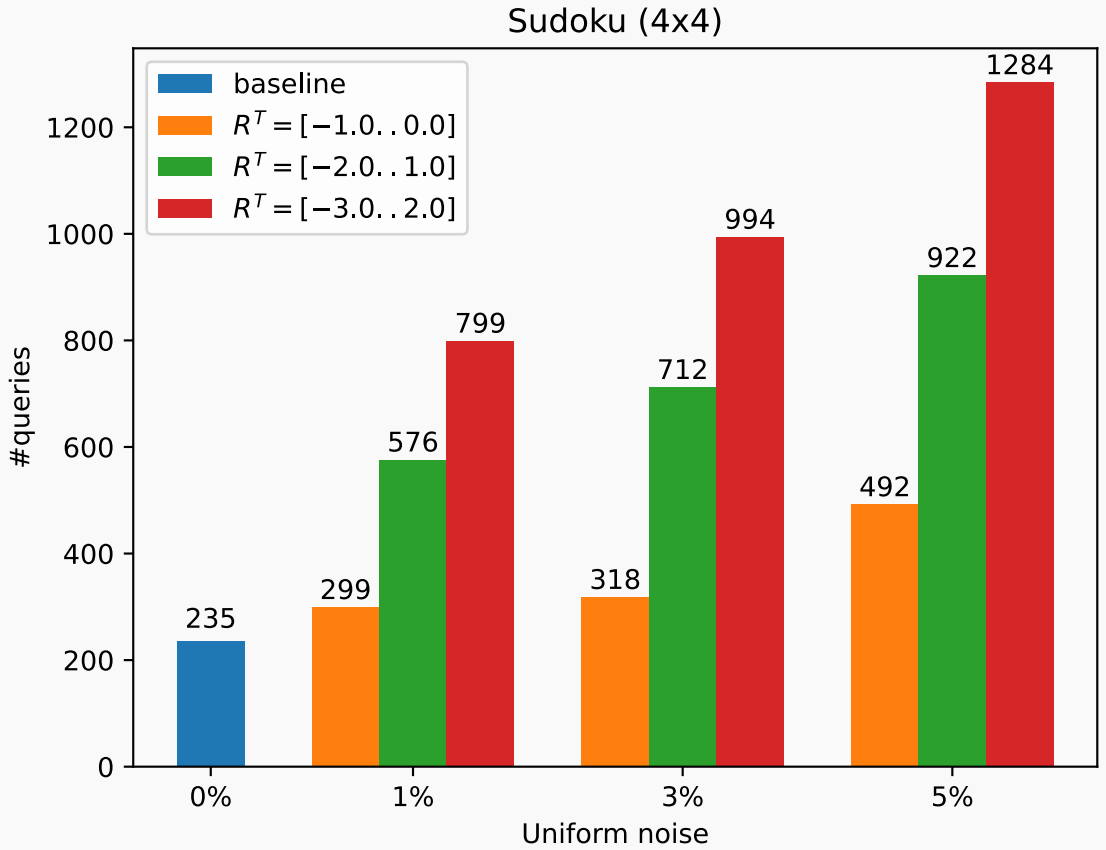
Small benchmark results



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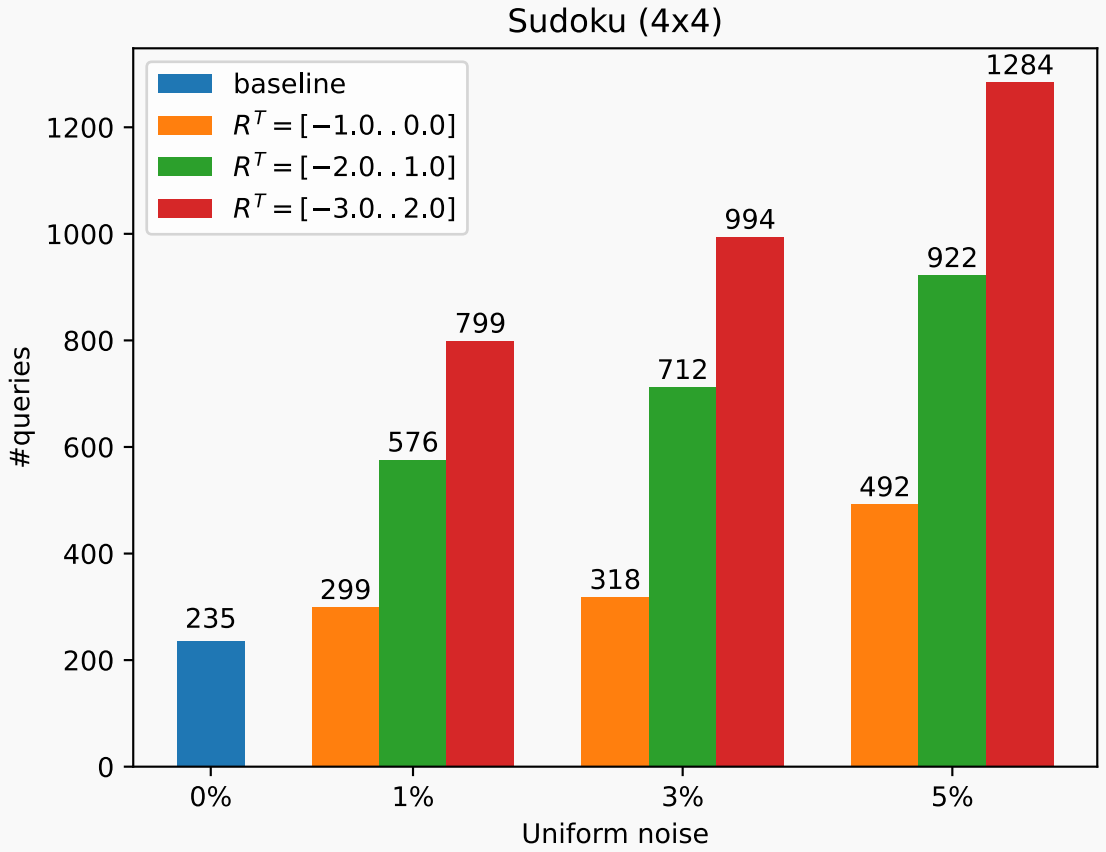
Small benchmark results



Unfortunately, the method fails to converge correctly on large benchmarks!

A more probabilistic direction

- Probabilistic interpretation of the evidence
- Accounting for structural evidence from ML
- Take inspiration from statistical learning like `SeqAcq`

Conclusion and discussion

- Active CA is paving a path towards the Holy Grail
  - Queries are (simpler) statements
  - But: noise-free assumption
- Robust active CA is possible but hard to scale
- An inexact setting perhaps requires inexact methods

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Bibliography

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[2] D. Angluin and D. K. Slonim, “Randomly fallible teachers: Learning monotone DNF with an incomplete membership oracle,” *Machine Learning*, vol. 14, no. 1, pp. 7–26, Jan. 1994, doi: 10.1007/BF00993160.

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