

Table Constraints for Integer Programming (CP'26)

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[Paper](#)



[Slides](#)



European Research Council



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Motivation

- *Integer Linear Programming* (ILP) solvers like Gurobi are extremely well-developed
- Solver-independent modelling languages (e.g. MiniZinc [7], CPMpy [4]) use ILP solvers to solve CP models
- But on CP instances, there is a performance gap between ILP and e.g. CP-SAT
 - ① Because important global constraints are not linearized as well as they can be
 - ② Because we do not use the full ILP toolset

Running example: `table([x,y,z], T)` with enforces the variables to take one row of T :

x	y	z
2	1	1
3	2	2
4	3	3
1	2	3
2	1	2

Integer encoding (INT)

- Standard encoding by Belov *et al.* [2] (default in MiniZinc)
- Binary variables $r_i \in \{0, 1\}$ for each row i , s.t. $r_i = 1$ iff row i chosen

Table T :

x	y	z
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3	2	2
4	3	3
1	2	3
2	1	2

Encoding:

$$\sum_{l \in 1..m} r_l = 1$$
$$\sum_{l \in 1..m} T_{l,i} r_l = x_i, \quad i \in 1..k$$

Example:

$$\begin{array}{rcccccc} r_1 & +r_2 & +r_3 & +r_4 & +r_5 & = & 1 \\ 2r_1 & +3r_2 & +4r_3 & +1r_4 & +2r_5 & = & x \\ 1r_1 & +2r_2 & +3r_3 & +2r_4 & +1r_5 & = & y \\ 1r_1 & +2r_2 & +3r_3 & +3r_4 & +2r_5 & = & z \end{array}$$

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Booleanized encoding (BOOL) [8]

We booleanize x with binary variables $\llbracket x = j \rrbracket \in \{0, 1\}, j \in D(x)$ such that $\llbracket x = j \rrbracket \Leftrightarrow x = j$:

$$\sum_{j \in D(x)} \llbracket x = j \rrbracket = 1, \quad \sum_{j \in D(x)} j \cdot \llbracket x = j \rrbracket = x$$

E.g. $\llbracket x = 1 \rrbracket + \llbracket x = 2 \rrbracket + \llbracket x = 3 \rrbracket + \llbracket x = 4 \rrbracket = 1, 1 \cdot \llbracket x = 1 \rrbracket + 2 \cdot \llbracket x = 2 \rrbracket + 3 \cdot \llbracket x = 3 \rrbracket + 4 \cdot \llbracket x = 4 \rrbracket = x$

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And booleanize T as a binary table $T^{\mathbb{B}}$:

x	y	z		$\llbracket x=1 \rrbracket$	$\llbracket x=2 \rrbracket$	$\llbracket x=3 \rrbracket$	$\llbracket x=4 \rrbracket$	$\llbracket y=1 \rrbracket$	$\llbracket y=2 \rrbracket$	$\llbracket y=3 \rrbracket$	$\llbracket z=1 \rrbracket$	$\llbracket z=2 \rrbracket$	$\llbracket z=3 \rrbracket$
2	1	1	$\equiv$	0	1	0	0	1	0	0	1	0	0
3	2	2		0	0	1	0	0	1	0	0	1	0
4	3	3		0	0	0	1	0	0	1	0	0	1
1	2	3		1	0	0	0	0	1	0	0	0	1
2	1	2		0	1	0	0	1	0	0	0	1	0

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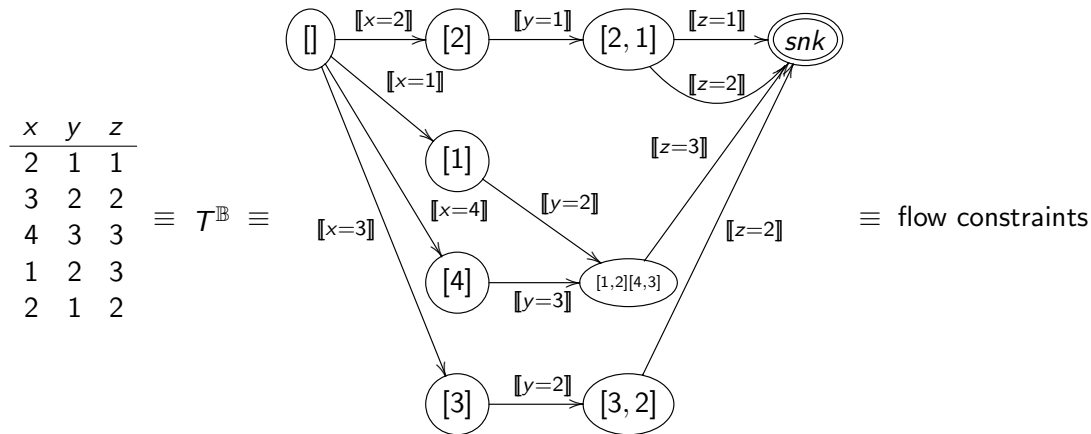
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2	1	2		0	1	0	0	1	0	0	0	1	0

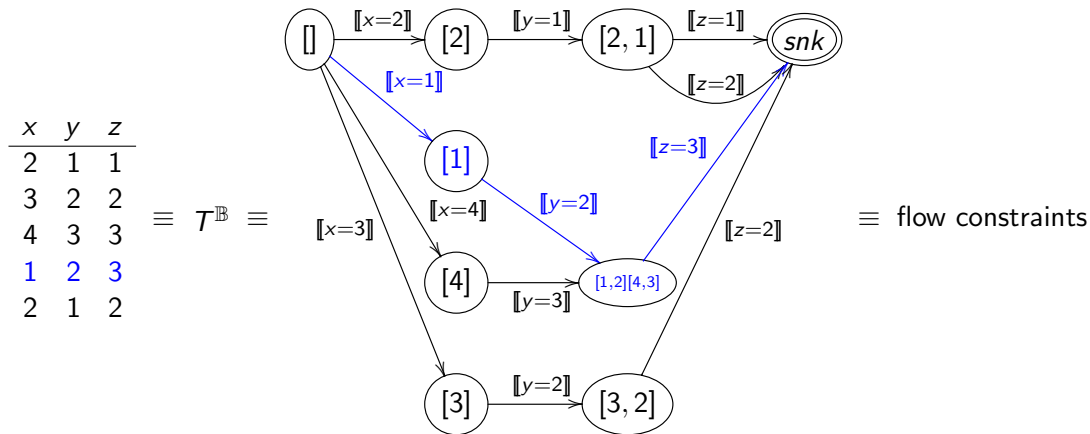
MDD-based Flow Encoding (MDD)

We can reformulate $T^{\mathbb{B}}$ as a reduced *Multi-valued Decision Diagram* (MDD) [3]



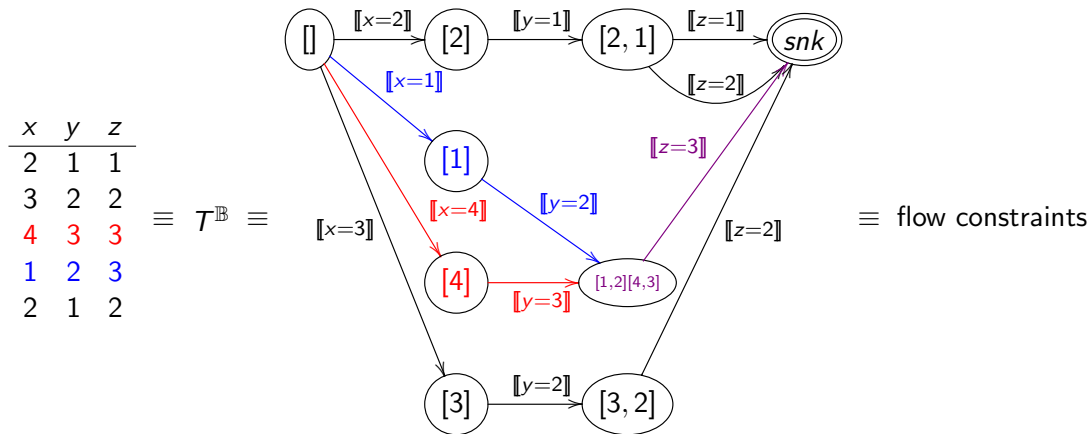
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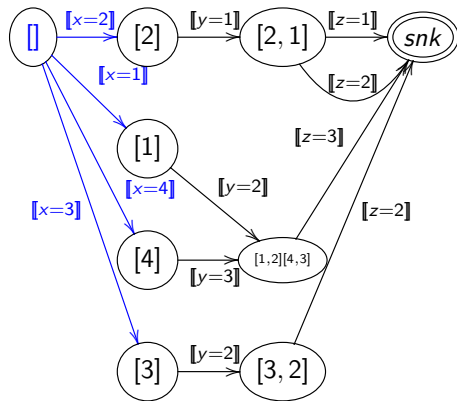


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MDD to flow constraints



Flow constraints:

$$\llbracket x = 1 \rrbracket + \llbracket x = 2 \rrbracket + \llbracket x = 3 \rrbracket + \llbracket x = 4 \rrbracket = 1 \quad \text{src}$$

$$\begin{aligned} \llbracket x = 1 \rrbracket &= e_{([1],2)} & [1] \\ e_{([1],2)} + \llbracket y = 3 \rrbracket &= \llbracket z = 3 \rrbracket & [1, 2][4, 3] \end{aligned}$$

$$\begin{aligned} \llbracket x = 2 \rrbracket &= \llbracket y = 1 \rrbracket & [2] \\ \llbracket y = 1 \rrbracket &= \llbracket z = 1 \rrbracket + e_{([2,1],2)} & [2, 1] \end{aligned}$$

$$\begin{aligned} \llbracket x = 4 \rrbracket &= \llbracket y = 3 \rrbracket & [4] \\ \llbracket x = 3 \rrbracket &= e_{([3],2)} & [3] \end{aligned}$$

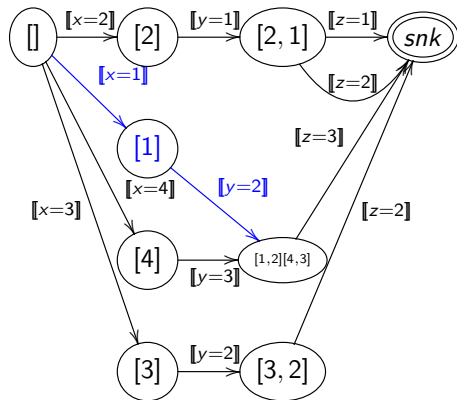
$$e_{([3],2)} = e_{([3,2],2)} \quad [3, 2]$$

$$e_{([2,1],2)} + \llbracket z = 1 \rrbracket + \llbracket z = 3 \rrbracket + e_{([3,2],2)} = 1 \quad \text{snk}$$

$$e_{([1],2)} + e_{([3],2)} = \llbracket y = 2 \rrbracket \quad \text{channel}$$

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$$[x = 2] = [y = 1] \quad [2]$$

$$[y = 1] = [z = 1] + e_{([2,1],2)} \quad [2, 1]$$

$$[x = 4] = [y = 3] \quad [4]$$

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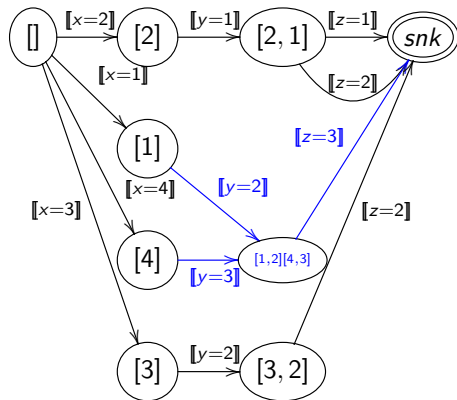
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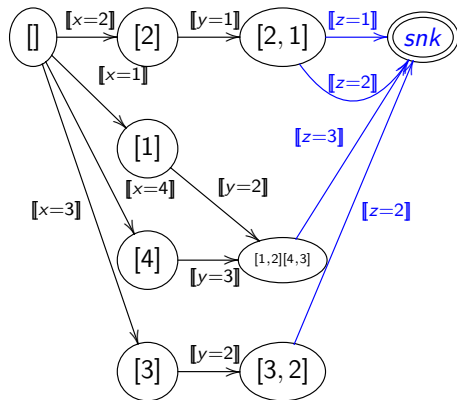
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Results: Encodings (CSP, 180 instances)

Benchmarks XCSP3 competition instances (2022–2025) [1], 180 CSP instances

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[illegible]

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INT	171		55	851.7	8249.3	–

POST	#instances posted	FEAS	#feasible instances found
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- ② If $\hat{v}$ violates a table, generate a **cut**
 - A valid cut violates $\hat{v}$, but does not violate the table constraint
 - E.g. $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 2 \rrbracket \leq 2$

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- ③ Repeat until $\hat{v}$ satisfies all tables

Can we generate a stronger cut, e.g. $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket \leq 1$?

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Can we generate a stronger cut, e.g. $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket \leq 1$?

Note: cuts with $\sum n$ unit terms $\leq n - 1$ are logical clauses, e.g.:

$$\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket \leq 1 \equiv \neg \llbracket x = 2 \rrbracket \vee \neg \llbracket y = 2 \rrbracket$$

GENERATE Example: $\hat{v} = (2, 2, 2)$

	$\llbracket x=1 \rrbracket$	$\llbracket x=2 \rrbracket$	$\llbracket x=3 \rrbracket$	$\llbracket x=4 \rrbracket$	$\llbracket y=1 \rrbracket$	$\llbracket y=2 \rrbracket$	$\llbracket y=3 \rrbracket$	$\llbracket z=1 \rrbracket$	$\llbracket z=2 \rrbracket$	$\llbracket z=3 \rrbracket$
1	0	1	0	0	1	0	0	1	0	0
2	0	0	1	0	0	1	0	0	1	0
3	0	0	0	1	0	0	1	0	0	1
4	1	0	0	0	0	1	0	0	0	1
5	0	1	0	0	1	0	0	0	1	0
$\hat{v}$	0	1	0	0	0	1	0	0	1	0

Cut: $0 \leq -1$ (all rows **violated**)

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1	0	1	0	0	1	0	0	1	0	0
2	0	0	1	0	0	1	0	0	1	0
3	0	0	0	1	0	0	1	0	0	1
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1	0	1	0	0	1	0	0	1	0	0
2	0	0	1	0	0	1	0	0	1	0
3	0	0	0	1	0	0	1	0	0	1
4	1	0	0	0	0	1	0	0	0	1
5	0	1	0	0	1	0	0	0	1	0
$\hat{v}$	0	1	0	0	0	1	0	0	1	0

Choose x , add $\llbracket x = 2 \rrbracket$: $\llbracket x = 2 \rrbracket \leq 0$ (rows 1 and 5 still violated)

GENERATE Example: $\hat{v} = (2, 2, 2)$

	$\llbracket x=1 \rrbracket$	$\llbracket x=2 \rrbracket$	$\llbracket x=3 \rrbracket$	$\llbracket x=4 \rrbracket$	$\llbracket y=1 \rrbracket$	$\llbracket y=2 \rrbracket$	$\llbracket y=3 \rrbracket$	$\llbracket z=1 \rrbracket$	$\llbracket z=2 \rrbracket$	$\llbracket z=3 \rrbracket$
1	0	1	0	0	1	0	0	1	0	0
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4	1	0	0	0	0	1	0	0	0	1
5	0	1	0	0	1	0	0	0	1	0
$\hat{v}$	0	1	0	0	0	1	0	0	1	0

Choose y , add $\llbracket y = 2 \rrbracket$: $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket \leq 1$

CUTLIFT Strengthen valid cut by **lifting** variables (*without* incrementing RHS)

Input: $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 2 \rrbracket \leq 2$

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Extensions: CUTLIFT and FRACTIONAL

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FRACTIONAL Generate cuts from **fractional** LP solutions

Input: $\hat{v} = [0, 0.5, 0.5, 0, 0, 1, 0, 0.5, 0.5, 0]$

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CUTLIFT Strengthen valid cut by **lifting** variables (*without* incrementing RHS)

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Input: $\hat{v} = [0, 0.5, 0.5, 0, 0, 1, 0, 0.5, 0.5, 0]$

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constraints	avg #constraints added	cuts	avg #cuts generated

Results: CSP (180 instances)

	POST	FEAS	<u>SOLVED</u>	time	constraints	cuts
INT	171		55	851.7	8249.3	–
BOOL	147		69	771.5	25670.4	–
MDD	154		67	780.8	38800.8	–
MDD-DOM	154		76	728.5	30884.2	–

POST	#instances posted	FEAS	#feasible instances found
<u>SOLVED</u>	#instances solved	time	PAR2 avg [s]
constraints	avg #constraints added	cuts	avg #cuts generated

Results: CSP (180 instances)

	POST	FEAS	<u>SOLVED</u>	time	constraints	cuts
INT	171		55	851.7	8249.3	–
BOOL	147		69	771.5	25670.4	–
MDD	154		67	780.8	38800.8	–
MDD-DOM	154		76	728.5	30884.2	–
GENERATE	174		36	988.6	8639.7	1857.5

POST	#instances posted	FEAS	#feasible instances found
<u>SOLVED</u>	#instances solved	time	PAR2 avg [s]
constraints	avg #constraints added	cuts	avg #cuts generated

Results: CSP (180 instances)

	POST	FEAS	<u>SOLVED</u>	time	constraints	cuts
INT	171		55	851.7	8249.3	–
BOOL	147		69	771.5	25670.4	–
MDD	154		67	780.8	38800.8	–
MDD-DOM	154		76	728.5	30884.2	–
GENERATE	174		36	988.6	8639.7	1857.5
FRACTIONAL	174		36	980.9	8639.7	3202.6

POST	#instances posted	FEAS	#feasible instances found
<u>SOLVED</u>	#instances solved	time	PAR2 avg [s]
constraints	avg #constraints added	cuts	avg #cuts generated

Results: CSP (180 instances)

	POST	FEAS	<u>SOLVED</u>	time	constraints	cuts
INT	171		55	851.7	8249.3	–
BOOL	147		69	771.5	25670.4	–
MDD	154		67	780.8	38800.8	–
MDD-DOM	154		76	728.5	30884.2	–
GENERATE	174		36	988.6	8639.7	1857.5
FRACTIONAL	174		36	980.9	8639.7	3202.6
CUTLIFT	174		45	925.0	8639.7	847.1
POST	#instances posted		FEAS	#feasible instances found		
<u>SOLVED</u>	#instances solved		time	PAR2 avg [s]		
constraints	avg	#constraints added	cuts	avg	#cuts generated	

Results: CSP (180 instances)

	POST	FEAS	<u>SOLVED</u>	time	constraints	cuts
INT	171		55	851.7	8249.3	–
BOOL	147		69	771.5	25670.4	–
MDD	154		67	780.8	38800.8	–
MDD-DOM	154		76	728.5	30884.2	–
GENERATE	174		36	988.6	8639.7	1857.5
FRACTIONAL	174		36	980.9	8639.7	3202.6
CUTLIFT	174		45	925.0	8639.7	847.1
ALL	174		55	871.5	8639.7	1326.7

POST	#instances posted	FEAS	#feasible instances found
<u>SOLVED</u>	#instances solved	time	PAR2 avg [s]
constraints	avg #constraints added	cuts	avg #cuts generated

Conclusion

Results: Eager vs. Lazy

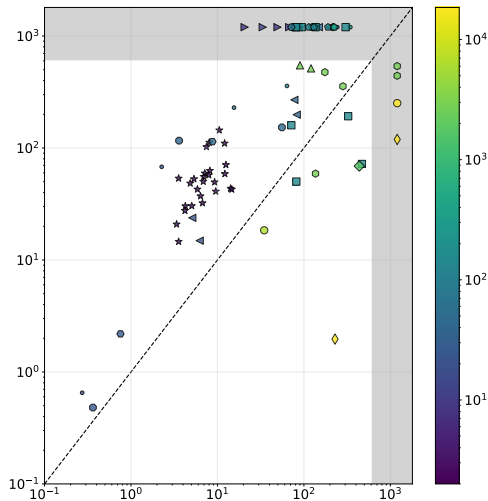


Figure: Solve time [s.]: Eager (top-left) vs. Lazy (bottom-right). Light colour = big table

Results (CSP, 180 instances)

	POST	FEAS	<u>SOLVED</u>	time	constraints	cuts
INT	171		55	851.7	8249.3	–
BOOL	147		69	771.5	25670.4	–
MDD	154		67	780.8	38800.8	–
MDD-DOM	154		76	728.5	30884.2	–
GENERATE	174		36	988.6	8639.7	1857.5
FRACTIONAL	174		36	980.9	8639.7	3202.6
CUTLIFT	174		45	925.0	8639.7	847.1
ALL	174		55	871.5	8639.7	1326.7
HYBRID (1000)	174		79	717.5	29635.4	49.9
HYBRID (2000)	174		80	715.6	29912.8	49.9
HYBRID (3000)	175		78	716.7	30243.3	37.3

POST	#instances posted	FEAS	#feasible instances found
<u>SOLVED</u>	#instances solved	time	PAR2 avg [s]
constraints	avg #constraints added	cuts	avg #cuts generated

Conclusion & Future Work

Summary:

- Studied eager encodings and lazy cuts for table constraints
- Novel MDD-based encodings outperform standard methods
- Cutlifting and fractional cuts improve cut generation
- Hybrid approach combines the strengths of both paradigms

Future work:

- Better understanding of which tables suit which method (e.g. sparsity)
- Extend to *negative, smart, and reified tables*



[Paper](#)



[Code](#)



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Example: CUTLIFT on $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 2 \rrbracket \leq 2$

- A **tight row** assigns variables such that the cut has LHS = RHS (e.g. row 2)
- A **tight column** is set to 1 in a tight row (e.g. $\llbracket x = 3 \rrbracket$)
 - $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 2 \rrbracket + \llbracket x = 3 \rrbracket \leq 2$ would violate row 2!

	$\llbracket x=1 \rrbracket$	$\llbracket x=2 \rrbracket$	$\llbracket x=3 \rrbracket$	$\llbracket x=4 \rrbracket$	$\llbracket y=1 \rrbracket$	$\llbracket y=2 \rrbracket$	$\llbracket y=3 \rrbracket$	$\llbracket z=1 \rrbracket$	$\llbracket z=2 \rrbracket$	$\llbracket z=3 \rrbracket$	<i>RS</i>
1	0	1	0	0	1	0	0	1	0	0	1
2	0	0	1	0	0	1	0	0	1	0	2
3	0	0	0	1	0	0	1	0	0	1	0
4	1	0	0	0	0	1	0	0	0	1	1
5	0	1	0	0	1	0	0	0	1	0	2

Cut: $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 2 \rrbracket \leq 2$

Example: CUTLIFT on $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 2 \rrbracket \leq 2$

- A **tight row** assigns variables such that the cut has LHS = RHS (e.g. row 2)
- A **tight column** is set to 1 in a tight row (e.g. $\llbracket x = 3 \rrbracket$)
 - $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 2 \rrbracket + \llbracket x = 3 \rrbracket \leq 2$ would violate row 2!

	$\llbracket x=1 \rrbracket$	$\llbracket x=2 \rrbracket$	$\llbracket x=3 \rrbracket$	$\llbracket x=4 \rrbracket$	$\llbracket y=1 \rrbracket$	$\llbracket y=2 \rrbracket$	$\llbracket y=3 \rrbracket$	$\llbracket z=1 \rrbracket$	$\llbracket z=2 \rrbracket$	$\llbracket z=3 \rrbracket$	<i>RS</i>
1	0	1	0	0	1	0	0	1	0	0	1
2	0	0	1	0	0	1	0	0	1	0	2
3	0	0	0	1	0	0	1	0	0	1	1
4	1	0	0	0	0	1	0	0	0	1	1
5	0	1	0	0	1	0	0	0	1	0	2

Lift $\llbracket x = 4 \rrbracket$: $\llbracket x = 2 \rrbracket + \llbracket x = 4 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 2 \rrbracket \leq 2$

Example: CUTLIFT on $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 2 \rrbracket \leq 2$

- A **tight row** assigns variables such that the cut has LHS = RHS (e.g. row 2)
- A **tight column** is set to 1 in a tight row (e.g. $\llbracket x = 3 \rrbracket$)
 - $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 2 \rrbracket + \llbracket x = 3 \rrbracket \leq 2$ would violate row 2!

	$\llbracket x=1 \rrbracket$	$\llbracket x=2 \rrbracket$	$\llbracket x=3 \rrbracket$	$\llbracket x=4 \rrbracket$	$\llbracket y=1 \rrbracket$	$\llbracket y=2 \rrbracket$	$\llbracket y=3 \rrbracket$	$\llbracket z=1 \rrbracket$	$\llbracket z=2 \rrbracket$	$\llbracket z=3 \rrbracket$	<i>RS</i>
1	0	1	0	0	1	0	0	1	0	0	1
2	0	0	1	0	0	1	0	0	1	0	2
3	0	0	0	1	0	0	1	0	0	1	2
4	1	0	0	0	0	1	0	0	0	1	1
5	0	1	0	0	1	0	0	0	1	0	2

Lift $\llbracket x = 4 \rrbracket$ (again): $\llbracket x = 2 \rrbracket + 2\llbracket x = 4 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 2 \rrbracket \leq 2$ row 3 now tight

Example: CUTLIFT on $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 2 \rrbracket \leq 2$

- A **tight row** assigns variables such that the cut has LHS = RHS (e.g. row 2)
- A **tight column** is set to 1 in a tight row (e.g. $\llbracket x = 3 \rrbracket$)
 - $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 2 \rrbracket + \llbracket x = 3 \rrbracket \leq 2$ would violate row 2!

	$\llbracket x=1 \rrbracket$	$\llbracket x=2 \rrbracket$	$\llbracket x=3 \rrbracket$	$\llbracket x=4 \rrbracket$	$\llbracket y=1 \rrbracket$	$\llbracket y=2 \rrbracket$	$\llbracket y=3 \rrbracket$	$\llbracket z=1 \rrbracket$	$\llbracket z=2 \rrbracket$	$\llbracket z=3 \rrbracket$	<i>RS</i>
1	0	1	0	0	1	0	0	1	0	0	1
2	0	0	1	0	0	1	0	0	1	0	2
3	0	0	0	1	0	0	1	0	0	1	2
4	1	0	0	0	0	1	0	0	0	1	2
5	0	1	0	0	1	0	0	0	1	0	2

Lift $\llbracket z = 1 \rrbracket$: $\llbracket x = 2 \rrbracket + 2\llbracket x = 4 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 1 \rrbracket + \llbracket z = 2 \rrbracket \leq 2$ row 1 now tight

Example: CUTLIFT on $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 2 \rrbracket \leq 2$

- A **tight row** assigns variables such that the cut has LHS = RHS (e.g. row 2)
- A **tight column** is set to 1 in a tight row (e.g. $\llbracket x = 3 \rrbracket$)
 - $\llbracket x = 2 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 2 \rrbracket + \llbracket x = 3 \rrbracket \leq 2$ would violate row 2!

	$\llbracket x=1 \rrbracket$	$\llbracket x=2 \rrbracket$	$\llbracket x=3 \rrbracket$	$\llbracket x=4 \rrbracket$	$\llbracket y=1 \rrbracket$	$\llbracket y=2 \rrbracket$	$\llbracket y=3 \rrbracket$	$\llbracket z=1 \rrbracket$	$\llbracket z=2 \rrbracket$	$\llbracket z=3 \rrbracket$	<i>RS</i>
1	0	1	0	0	1	0	0	1	0	0	2
2	0	0	1	0	0	1	0	0	1	0	2
3	0	0	0	1	0	0	1	0	0	1	2
4	1	0	0	0	0	1	0	0	0	1	2
5	0	1	0	0	1	0	0	0	1	0	2

Lift $\llbracket x = 1 \rrbracket$: $\llbracket x = 1 \rrbracket + \llbracket x = 2 \rrbracket + 2\llbracket x = 4 \rrbracket + \llbracket y = 2 \rrbracket + \llbracket z = 1 \rrbracket + \llbracket z = 2 \rrbracket \leq 2$ all rows tight